QUARK EFFECTS IN NUCLEI

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By

MADHULIKA MEHROTRA

M.Sc.

Under the supervision of

PROF. (Mrs.) INDIRA MEHROTRA

Department of Physics

University of Allahabad, Allahabad

DEPARTMENT OF PHYSICS

UNIVERSITY OF ALLAHABAD

ALLAHABAD - 211002 INDIA

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DECLARATION

I declare that the thesis entitled *"Quark Effects in Nuclei"* is my own work conducted under the supervision of ***Prof. (Mrs.) Indira Mehrotra, Department of Physics, University of Allahabad, Allahabad,*** approved by Research Degree Committee. I have put in prescribed period of attendance with the supervisor in the department.

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I. Mehrotra

Signature of the Supervisor

[Prof. (Mrs.) INDIRA MEHROTRA]

Madhulika. Mehrotra.

Signature of the Candidate

[MADHULIKA MEHROTRA]

Signature of HOD

[PROF. G.K. PANDEY]

UNIVERSITY OF ALLAHABAD

Prof. (Mrs.) Indira Mehrotra

Physics Department
Allahabad - 211002
INDIA
E.Mail: indum81@rediff.com

CERTIFICATE

This is to certify that the work entitled *"Quark Effects in Nuclei"* is a piece of research work done by *Mrs. Madhulika Mehrotra,* under my guidance and supervision for the *Degree of Doctor of Philosophy of University of Allahabad, Allahabad, INDIA*. The candidate has put in prescribed period of attendance with me.

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[Prof. G. K. Pandey]

IMehrotra

Signature of the Supervisor

[Prof. (Mrs.) Indira Mehrotra]

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[2] **The Schmidt Diagram and Quark Degree of Freedom.**

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[3] **Quark Degrees of Freedom in the Binding Energy Difference of $^{14}_{\Lambda}C \sim {}^{14}_{\Lambda}N$.**

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nucleons $(P_{NN}^{6q}(r_0))$ and hyperon $(P_{\Lambda N}^{6q}(r_0))$for different values of the cut off radius r_0. $P_{Q_1}^{6q}(r_0)$ and $P_{Q_2}^{6q}(r_0)$ are the six-quark and nine-quark bag formation probabilities with one and two core nucleons respectively. ΔV_c is the loss in Coulomb energy when the two protons overlap for $r < r_0$. The values shown have been calculated with the parameters of Gal I using Moshinsky method. (In Gal I $\nu_N = 0.41 fm^{-2}$; $\nu_\Lambda = 0.49 fm^{-2}$)

values marked with † are obtained with $\mu'_{\Lambda} = -1.08\,\mu_{\Lambda} = -0.662\ n.m.$

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1. INTRODUCTION

1

INTRODUCTION

The question" what are the actual constituents of atomic nucleus ?" has continuously been asked since the very beginning of nuclear research, early in this century. Our ideas about the nuclear constituents, and consequently those of the nuclear matter, have been determined by the current understanding of elementary particles. From the time when neutrons and protons were treated as elementary particles and as such where point like, to the present day status, when the nucleons are made up of quarks, there has been a considerable change of our ideas on the nuclear constituents. Although Yukawa theory of boson phenomenology of the nucleon–nucleon interaction is nearly half a century old, nuclear forces are still not understood from a more fundemental point of view. This understanding must ultimately come from quantum chromodynamics (QCD), the theory of strong interactions of quarks and gluons, since neither nucleons nor pions are "elementary". Thus invoking quark degrees of freedom in nuclei has a far reaching consequence on several nuclear properties particularly those associated with the short-distance part of the nucleon-nucleon interaction.

1.1 The Conventional Picture

The nucleus has been treated as a system of nucleons (protons and neutrons) bound by the strong interaction. The interaction is usually

expressed in terms of a two-body potential between nucleons. There have been several kinds of potentials obtained phenomenologically or microscopically by fitting the nucleon-nucleon (NN) scattering data and the observed deuteron properties. For a description of the long-range part of the potential, it is well established that the one pion exchange potential (OPEP) plays the most important role. There are, however, some differences among different potentials in the medium and short-range parts of the potential. The NN scattering data suggests a strong repulsive interaction in the high-energy region $(E_{Lab} \geq 250MeV)$. Therefore, phenomenologial potentials , like the Hamada-Johnston (*Hamada and Johnston*, 1962) and Reid potentials (*Reid,* 1968) assume a hard core or soft core at short distances to describe the short-range repulsion. In a microscopic description of the two-body potential, like the one boson exchange potential (OBEP), heavy vector mesons such as the ω–meson (isoscaler) and ρ–meson (isovector) are responsible for the short-range repulsion. The Paris potential (*Lacombe et al*, 1980) which has been widely employed recently assumes an energy dependent short-range repulsion. So far these potentials have been employed as a basic ingredient in the study of nuclear many-body problems and conventional nuclear structure models have met with great success in explaining many experimental data.

It is now well known that the nucleon itself has a finite size and internal structure. The measured rms radius of proton is 0.9fm (*Brokowski et al*, 1974;*Weise,* 1984). The extended size of nucleons can be thought of due to boson cloud surrounding the nucleon core. This point is taken into account in terms of nucleon form factor when the electromagnetic and weak interation properties are discussed. In the meson exchange potential the finite size effect is taken into account by introducing a form factor for the meson nucleon vertex.

1.2 Quark Structure

In 1964 Gell-Mann (*Gell-Mann*, 1964) and Zweig (*Zweig*, 1964) independently proposed that hadrons are composite particles with quarks as constituents. At nearly the same time a large number of elementary particles had been found. These particles grouped themselves into irreducible multiplets of the group SU(3) and with the introduction of a fundamental SU(3) triplet (the quark flavors u, d, s), one was able to explain the multiplet structure of the observed baryons and mesons. The earliest experimental support for this came from a number of deep inelastic scattering (DIS) experiments of leptons and neutrinos off protons. In these experiments at Stanford in the late sixties, high-energy electrons were scattered off protons, and one observed that there were more scattered electrons with high transverse momenta than if the proton has been a diffused distribution of matter. Furthermore, the energy and angular distributions of the scattered electrons indicated scattering from "pointlike" particles or partons. Thus the quarks (and "gluons") were discovered in experiments analogus to the famous Rutherford experiments that revealed the atomic nucleus.

Quarks cannot be isolated. In the experiment one can see them as "free" particles only if one uses a resolution better than 1GeV/c. At distances of 1fm quarks hadronize. At small scales (1GeV/c), QCD seems to work well (*Wilczek,* 1982; *Söding and Wolf*, 1981) and is generally accepted as being the theory of the strong interaction. At large scales the coupling constant increases and the perturbative methods of QCD does not work (*Yazaki,* 1995). But this is just the domain important for nuclear physics. The nuclear force is no longer the fundamental force of the strong interaction; nucleons and pions are composite particles. The field of the strong interaction is not the pion but the gluon field coupled to the color of

the quarks. The nuclear force is just a remainder of the strong force of the color neutral nucleon.

Following the success of the description of single hadron properties in terms of quark model, there has been many attempts to study the intraction between nucleons and hadrons in general within the quark model. The long-range interaction can be well described by the pion exchange. In the region where two hardrons overlap each other, the internal structure of hadrons is expected to play more explicit role. Therefore, study of hadron-hadron interaction within the quark model concerns the short- and medium-range part of the interaction of the baryon-baryon interaction (*Shimizu,* 1989). In the quark model due to Pauli principle at the quark level various quark exchange processes between the three quark clusters (nucleons) occur. For example, a gluon or a pion and a quark are simultaneously exchanged between two nucleons. These gluon-quark (*Faessler et al,* 1982, 1983; *Shimizu*, 1989). and pion-quark (*Bräuer et al,* 1985; *Fernandez and Oset,* 1986) exchange processes can produce a short range repulsion in the *NN* interaction.

When the quark concept was first introduced, it was only thought of as a convenient mathematical tool for explaining classification properties of elementary particles. But now, following the modern development of guage theories and the advent of QCD, the quark is no longer restricted to the domain of elementary particle physics, but reaches other field such as nuclear physics. During last few years a great deal of attention has been paid to the studies of the quark degrees of freedom in nuclei and it has led to a better understanding of several nuclear phenomenon.

1.3 Quark Effects in Nuclei

One of the most interguing question concerning nuclei is the effect of invoking quark degrees of freedom in nuclei on various nuclear properties. The relatively great success of the conventional nuclear model might suggest the lack of any direct evidence for the presence of quarks in nuclei. This however is not true. Since quarks are mostly confined in nucleons, there influence will be evident only at very short distances and very high energies. Furthermore, since quark effects have not yet been noticed in conventional measurments, they must be small at best and must be studied under circumstances where competing phenomenon from other processes are known to precision better than the anticipated contributions from quarks. Large number of experiments have been performed centering round the EMC effect, first noted in 1983 by the European Muon Collaboration (*Aubert et al,* 1983, 1987). These experiments involve the deep-inelastic scattering of leptons or nucleons on nuclei for $Q > 1(GeV/c)^2$.The results of these experiments show that the structure function per nucleon in iron differs significantly from that in deuterium. As the binding of two nucleons in deuteron is extremly weak it can be neglected when compared with the binding of heavier nucleus, and to a good approximation deuteron is just a free proton and a free neutron. Thus the ratio of the two structure functions demonstrates the influence of nuclear binding on the momentum distribution of the quarks in the nucleons. It turns out that the quark momentum distribution is strongly changed in nuclei. Thus the results of these experiments support the conviction that even though quarks cannot be isolated, their properties can be determined experimentally.

With the EMC effect providing a sound experimental footing for the quark picture, several groups have searched for quark effects on various other nuclear properties like, the short range part of the NN interaction,

the nuclear magnetic moments, reactions on various nuclear targets, etc. Since a microscopic treatment in terms of quark degrees of freedom is not feasible for nuclei with A>2, various quark models have been used as the starting point in most of these studies.

The quark effects in the NN scattering (*Liberman*, 1977; *De Tar*, 1978; *Ohta et al*, 1982) and electron nucleus scattering *(Buchmann et al* 1989; *Oka and Yazaki,* 1983; *Takeuchi et al;* 1986; *Yamauchi et al,* 1989) has been investigated in the quark cluster model. The main feature of this model is that, due to the Pauli principle on the quark level, various quark exchange processes between the three-quark clusters (nucleons) occur. Exchange of quark-gluon-quark (*Shimizu,* 1985; *Bräuer et al,* 1985; *Fernandez and Oset,* 1986) gives rise to additional electromagnetic currents. These currents are called the quark exchange currents and generate the short range repulsion between the nucleons. Thus, these currents are a consequence of the underlying quark degrees of freedom and represent the quark effects in the electromagnetic properties of the nucleus. The electron scattering at high momentum transfers magnifies the short-range part of the nuclear wave function which in turn is related to the short-range part of the NN interaction.

All reactions involving strongly interacting particles should conserve parity. However in an experiment (*Nagle et al,* 1978; *Balzer,* 1980) conducted at los Alamos, it has been observed that there is a small amount of parity violation in proton-proton scattering. The parity violation can be defined as an asymmetry parameter A,

$$A = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}$$

where σ_+ and σ_- refer to spin up and spin down states. If parity is conserved the quantity A vanishes. The experimental value of A ranges between A = $1.7\pm0.8\times10^{-7}$ to $2.3 \pm 0.8 \times 10^{-7}$. The parity violation amplitude is generated by W and Z boson exchanges between quarks. The relevant fermions are quarks, not nucleons.

The role of quarks in understanding the $\pi^+ d \Leftrightarrow pp$ reaction is also important (*Miller and Kisslinger,* 1983). In the conventional approach the momentum transfer in the reaction is accomplished by the exchange of a virtual π or ρ meson. These studies have enjoyed a good deal of success and explained many features of the $\pi^+ d \leftrightarrow pp$ data, but for large proton energies, the discrepancy between the theoretical and experimental results is nearly 20%. If a pion of kinetic energy 180MeV is absorbed on a deuteron, energy momentum conservation in the center-of-mass frame requires that each outgoing nucleon has a kinetic energy of 140MeV (or momentum 550 MeV/c). As the deuteron is a weakly bound system, the $\pi^+ d \Leftrightarrow pp$ process involves a momentum transfer of about 550MeV/c to a nucleon. Estimating the relevant distance scale from the uncertainty principle leads to h/500 MeV/c = 0.4fm. This is smaller than the radius of a nucleon (~1fm), so that nucleons are expected to overlap during the $\pi^+ d \rightarrow pp$ process. If the composite nature of baryons is relevant, it is natural to invoke quark degrees of freedom.The short distance contribution arising from quark degrees of freedom have large influence on the computed $pp \rightarrow d\pi^+$ cross section data in the right direction.

Quark effects are also important in the study of deuteron, the most loosely bound nucleus. One of the important parameters in the study of deuteron is the asymptotic ratio, η, of D to S state wavefunctions. As an asymptotic quantity, it is an observable. The experimental results (*Bordely*

et al, 1982) show that $\eta_{\exp} = 0.0271 \pm 0.0004$. *Ericson and Rosa-Clot* (1982, 1983) using ordinary potential theory have shown that experimental value of η can be accounted for if the size of the nucleon bag is less than or equal to about 0.6 fm, a value rather too small from the point of view of bag model. If the nucleons are treated as composite structure of quarks the tensor forces due to pions and gluons exchanged between quarks increase the size of the nucleonic bag in the more acceptable range of 0.8 to 1.1fm (*Guichon and Miller,* 1984) . For nucleonic bag of radius 1fm the quark contribution to η is about 6%. This is a large percentage of the asymtotic property of most loosely bound nucleus and indicates that quark effects may be very significant in the more tight nuclei. The role of six-quark cluster in the proton-deuteron breakup reaction (*Deloff and Siemiarczuk,* 1986) and deuteron electrodisintegration (*Cheng and Kisslinger,* 1986) show that six-quark cluster effect becomes important in the region of high momentum transfer.

Using the nonrelativistic quark model of the nucleons and considering the quark exchange between atmost two nucleons, *Modarres* (1994) has calculated the quark distributions in the ground states for doubly magic nuclei (^4He, ^{16}O and ^{40}Ca) and has observed that quark exchange and the short-range repulsion arising from NN correlation reduce the quark density at short distances. Quark effects have also been studied for pion absorption (*Miller and Kisslinger,* 1983), weak p-p asymmetry (*Kisslinger and Miller,* 1983), the charge form factors of ^3He and ^3H (*Kisslinger,* 1982; *Henley et al,* 1983), non-leptonic decay of Λ–hypernuclei (*Cheung et al,* 1983; *Heddle and Kisslinger,* 1986) and the pion double charge exchange reaction $\left(\pi^+,\pi^-\right)$(*Miller,* 1984; *Johnson and Kisslinger,* 1986).

1.4 Problems for Present Work

The above discussion indicates that there is an evident possibility of improving the understanding of the nuclear properties by incorporating explicitly the quark degrees of freedom in nuclei. The strong interaction that holds the nucleus together has its ultimate origin in the quark and gluon interactions provided by QCD. This being hard to handle, most of these attempts are based on QCD motivated phenominology rather than QCD itself. The essential underlying philosophy is that at short distances, the NN system should be described by six-quark cluster. In the present work a hybrid quark nucleon model is used to investigate the quark effects in the binding energy and magnetic moments of some finite nuclei.

1.4a Binding Energy

The study of binding energy difference between mirror nuclei has been a subject of interest since long (*Nolen and Schiffer*, 1969; *Bluden and Iqbal*, 1987; *Brandenburg et al*, 1988; *Hatsuda et al*, 1990). With the availability of reliable experimental data on Λ-binding energies such studies have also been extended to mirror hypernuclei (*Friar and Gibson*, 1978; *Gibson and Lehman*, 1979). The most important contribution to this binding energy difference of mirror pair originates from the Coulomb energy, but as is well known the Coulomb energy contribution is not sufficient to explain the observed difference in the energy. The discrepancy which is non Coulombic in origin is known as Nolen-Schiffer (NS) anomaly (*Nolen and Schiffer*, 1969). In the traditional nuclear theories origin of NS anomaly has been attributed to different aspects of theoretical charge symmetry breaking NN interaction (*Miller*, 1990) or ΛN interaction (*Biswas et al*, 1980).With the realization that strong nucleon-nucleon interaction has its

ultimate origin in quark and gluon interaction provided by quantum chromodynamics, an alternative approach has emerged to understand the NS anomaly through incorporating explicit quark degrees of freedom in the nuclear wave functions. In the present work we have studied the quark contribution to the binding energy difference of p-shell hypernuclei pair ${}^{6}_{\Lambda}He \sim {}^{6}_{\Lambda}Li$ and ${}^{14}_{\Lambda}C \sim {}^{14}_{\Lambda}N$ in hybrid quark model framework. *Nag and Sural* (1992) have studied the quark contribution to the Λ-binding energy in s-shell hypernuclei pair ${}^{4}_{\Lambda}He \sim {}^{4}_{\Lambda}H$. Experimental binding energy of Λ-particle is more in ${}^{4}_{\Lambda}He$ (proton rich partner) than in ${}^{4}_{\Lambda}H$ (neutron rich partner) by nearly 360 keV. Earlier studies in this direction are of *Greban and Thomas* (1984), *Köch and Miller* (1985), and *Wang et al* (1988). In the study of *Greban and Thomas* (1984) and *Köch and Miller* (1985) contribution of six-quark cluster formation of the overlapping nucleons on the binding energy difference of mirror nuclei with A = 3,13,17,29,33,41 has been estimated. These authors have observed that the quark effect is in the right direction to account for the missing Coulomb energy problem. In an alternative approach *Wang et al.* (1988) have studied the binding energy differece for mirror nuclei with A = 3, 5, 11, 13, 15, in both nonrelativistic and relativistic quark models based on the one-gluon exchange interaction. They have made calculations in both the six-quark and resonating-group method to treat the effects of overlapping nucleons in nuclei. These authors have observed that nonrelativistic potential models tend to give Nolen-Schiffer anomalies larger than experimental results, while bag models give too small results. The resonating-group method is much more complete but complicated compared to the six-quark method.

Our calculations are based on the hybrid quark model (HQM) employed in the earlier studies. According to this model two nucleons maintain their identity as long as the distance between them is greater than a certain cutoff radius r_0. For distances smaller than r_0 the two

baryons overlap and form a six-quark bag. The quark contribution to the binding energy difference depends on (a) the probability of the formation of six-quark bags and (b) the energy difference between the six-quark bags of overlapping nucleons (or hyperon and nucleon) than that of isolated nucleons (or hyperon and nucleon). In both ${}^{6}_{\Lambda}He \sim {}^{6}_{\Lambda}Li$ and ${}^{14}_{\Lambda}C \sim {}^{14}_{\Lambda}N$ there is a valence nucleon outside a closed core of nucleons and Λ-particle. We have calculated the six-quark bag probability of the valence particle with the core nucleon and the corresponding contribution to the binding energy difference of the mirror pair ${}^{6}_{\Lambda}He \sim {}^{6}_{\Lambda}Li$ and ${}^{14}_{\Lambda}C \sim {}^{14}_{\Lambda}N$. We have also estimated the six-quark cluster formation of the valence nucleon with the hyperon and its effect on the binding energy difference. ΛN bag formation effects are known to make significant contribution to the Λ-nonmesonic decay rates in the finite nuclei (*Heddle and Kisslinger,* 1986) . This effect was not included in the work of *Nag and Sural* (1992).

1.4b Magnetic Moments

In the conventional nuclear theory the nuclear shell model works well for the calculation of nuclear magnetic dipole moments of nuclei. In the independent particle model approach magnetic moments of nuclei with configuration of closed core+1 nucleon are given by Schmidt values (*Tanaka,* 1989). The deviation of these values from the experimental data has been a subject of continuous interest since long. A number of corrections to the independent particle model values of the magnetic moments have been considered in the literature which can contribute to this discrepancy. *Arima* and *Horie* (1954) and independently *Blin-Stoyle* (1953) have attributed the discrepancy to the mixing of the configurations in the shell model states. It is expected that different configurations of the nucleons, become intermingled under short range attractive forces and can cause deviations from the single particle values. On the other hand

some authors (*Harper et al,* 1972; *Hyuga et al,* 1980; *Bhaduri et al,* 1986; *Tomusiak et al,* 1985) have attributed the discrepancy to meson exchange current in the two nucleon interaction, which includes various contributions arising from seagull terms, pions and delta isobar current terms, etc.

In some of the earlier studies core polarization (particle-hole excitation) has been introduced as the conventional correction to the independent particle shell model values. If the core polarization effects are treated perturbatively they can be estimated by defining effective operators (*de-Shalit,* 1961; *Talmi,* 1972) in the framework of Rayleigh-Schrödinger perturbation theory. In a recent study *Kitagawa* (1999) has estimated the correction to the magnetic moment of p-shell nuclei, arising due to core polarization and also due to valence polarization. The results of these studies show that various corrections arising due to configuration mixing effect, meson exchange current and core polarization are important but not sufficient to explain the deviation of experimental data from the Schmidt values and thus one can look for corrections arising from additional degrees of freedom in nuclei. It is now well established that nucleons are composite particles consisting of quarks. Thus the internal structure of nucleons can make significant changes in the nuclear properties.

In the work of *Yamauchi et al* (1991) the quark degrees of freedom have been taken into account by introducing effective quark exchange current operators, which are electromagnetic current operators on the quark levels. Apart from the usual impulse and meson exchange currents, these effective currents arising due to quark degrees of freedom contain new terms which are generated by the Pauli principle at the quark level.

These authors have investigated the role of additional quark exchange currents in the magnetic moments of several closed shell$\pm$1 nuclei.

In a somewhat similar approach *Takeuchi and Shimizu* (1986) have estimated the effect of quark exchange currents on the magnetic moments of light hypernuclei and Gamow-Teller type β-decay matrix elements and have observed that these effects cause quenching in the free particle values.

In an alternative approach *Radhey Shyam et al* (1988) have considered the possibility of formation of six-quark matter by overlapping of nucleons. The nucleons have a radius of 0.9fm and such large nucleons can be expected to partly overlap in the nucleus and form a cluster of six-quarks. This new picture of overlapping nucleons and forming a six-quark bag can contribute significantly towards deviations of the magnetic moments, from the corresponding single particle values. These authors following the HQM approach have shown that experimental values of magnetic moments of deuteron can be explained if it has an admixture of six-quark states with probability ranging between 3 to 6% for different types of nucleon-nucleon interaction.

In the present work we have analysed the effect of additional quark degrees of freedom in nucleons on the magnetic moments of nuclei with closed shell+one nucleon. We have estimated the correction to the magnetic moment of mirror pair of nuclei with A = 13, 17, 29, 33, 41, in the framework of HQM discussed earlier.

The effect of quark structure of nucleons on nuclear properties has been studied by certain other methods as well. *Abbas* (1987) has shown that good fits to the experimental data on the magnetic moments of ^3H-^3He pair

can be obtained if nucleonic bags consisting of three quarks are considered to be deformed. In the present work we have estimated the deformation parameters of nucleonic bags for mirror pair of nuclei with A = 13, 17, 29, 33, 41 which reproduce the observed experimental values.

Theoretical studies of magnetic moments of nuclei have been an aid in understanding nuclear structure and nuclear interactions. Therefore, the study of magnetic properties of hypernuclei in addition to their binding energy calculations is useful in understanding such systems. As the experimental data on the magnetic moments of hypernuclei is still awaited, only few theoretical studies have been carried out in the past. *Nag* (1985) has estimated the magnetic moment of the hypertriton treating it as a three body problem with a purely central ΛN interaction and central+tensor NN interaction. The tensor interaction introduces an admixture of D-state wavefunction in the predominant spatially symmetric ^2S state. The predicted magnetic moments of hypertriton dependens on the amount of D-state probability and also on the choice of ΛN and NN interactions. Nuclear structure studies of the hypernuclei has been carried out by various authors in either of the two complementary approaches, the shell-model or the cluster-model (*Gal et al,* 1978; *Tong and Herndon,* 1966; *Bando et al,* 1982 *and Motoba et al,* 1984). In making a detailed study of light p-shell hypernuclei *Motoba et al* (1985) have observed that both these aspects are crucially important and the incorporation of the two ingredients can be achieved by the microscopic treatment for the three clusters α, x and Λ, where x is a nucleon or a cluster of three nucleons. In the earlier study of *Bodmer et al* (1984) clusters are treated as structureless particles whereas in the work of *Motoba et al,* (1985) the α and x clusters are treated to be composite and the antisymmetrization among all nucleons are taken into account properly. These authors have computed the magnetic moments of p-shell hypernuclei using well-developed cluster wavefunctions and

important shell-model configurations without any spurious center-of-mass excitation. *Tanaka* (1989a) has used the effective moment method to calculate the magnetic moments of light hypernuclei with $p^m n^k Y\ (Y = \Lambda, \Sigma \text{ and } \Xi)$ configuration assuming shell model wavefunction with good isospin. These authors have predicted the magnetic moments of hypernuclei using known magnetic moments of nuclei and hyperon, and have obtained results similar to those of *Motoba et al* (1985). In a later work *Tanaka* (1989b) has obtained the Schmidt diagram for magnetic moments of light hypernuclei with the *X–N–Y* configuration, where *X, N* and *Y* denote a nuclear core $\left({}^{4}He,\ {}^{16}O \text{ and } {}^{40}Ca\right)$, a nucleon ($n$ and p) and a hyperon (Λ, Σ and Ξ)respectively. Using shell model wavefunction with good isospin he has investigated the hyperon-induced configuration mixing effects on magnetic moments of Λ–hypernuclei and has observed that these corrections are small in many cases, except for the special case of ${}^{16}_{\Lambda}N$ and ${}^{40}_{\Lambda}K$.

In the last decade a number of studies of the various characteristics of the nuclear ground state and of the nuclear dynamics have been made in the relativistic mean field models (*Gambhir et al,* 1990; and references therein). These models based on Dirac equation with strong scaler and vecter potentials were originally designed for spherical systems and later extended to deformed ones (*Hofmann* and *Ring,* 1988 and *Furnstahl* and *Price*, 1989). In particular, attention has been focused on areas for which relativistic predictions differ significantly from those obtained in the traditional framework of nonrelativistic theory, for example the study of the nuclear currents and magnetic moments in particular. Both the relativistic and nonrelativistic models yield similar predictions for the isoscalar nuclear magnetic moments. While the nonrelativistic (Schmidt) values are obtained directly in a single-particle approach, in the relativistic mean field calculations the result comes from a compensation of two effects, the

enhancement of the valence particle current due to the reduction of the effective nucleon mass and the contribution of the additional current from polarized core nucleons (*Matsui,* 1981; *Bentz et al* , 1985 and *Mc Neil et al,* 1986). In the case of hypernuclei, enhancement of the current caused by the valence hyperon is not entirely cancelled by the core corrections, since the mass of the hyperon and the coupling constants are different from those of the nucleon. Moreover, the Λ single-particle current does not contribute to the hypernuclear magnetic moment (Λ is a neutral particle) and the Schmidt value is entirely due to the anomalous moment of the Λ-hyperon. But the nonvanishing current from the polarized core is the source of significant deviation from the Schmidt values. *Cohen et al* (1987) have studied the magnetic moments of ${}^{13}_{\Lambda}C$, ${}^{17}_{\Lambda}O$, ${}^{41}_{\Lambda}Ca$, ${}^{91}_{\Lambda}Zr$, ${}^{209}_{\Lambda}Pb$ in the mean field approximation to an extended σ–ω (Walecka) model and has estimated the values of magnetic moments in a spherical core where core response to the added particle is calculated in the random phase approximation. *Mares* and *Zofka* (1990) have also estimated the magnetic moments of the same set of hypernuclei in the mean field model with a deformed core. Both these studies predict similar and rather sizable corrections to the Schmidt values ranging from 8 to 12%. As explained above in the case of Λ-hyperon orbiting a nucleus, the nucleon valence particle current is not totally cancelled as the coupling of the valence particle Λ to the vector meson is different than that for the nucleons. However these calculations omitted the very important tensor coupling of the vector field (ω) to the Λ-hyperon (*Dover and Gal, 1984* and *Jennings,* 1990). This change in the Λ-coupling induces a modification in the sector of hyperon-baryonic current and has implications on the structure of the backflow current for finite nuclei.

In the later studies *Gottane et al* (1991) and *Cohen* and *Noble* (1992) have calculated the magnetic moments of several hypernuclei within the framwork of σ-ω model and have shown that the inclusion of tenser term to

describe the strong σ-ω coupling restores the magnetic moments to values very close to the Schmidt limit. *Dover et al* (1995) have shown that Λ-Σ mixing due to the strong $\Lambda N \leftrightarrow \Sigma N$ interaction can also produce deviation of hypernuclear magnetic moments from the Schmidt values. These authors have also observed that Σ–Λ mixing becomes more effective when there is no change in the nuclear core states and deviation from Schmidt values are larger for p-shell hypernuclei compared to that of the s-shell hypernuclei.

In the present work me have calculated the magnetic moments of the follwing two classes of hypernuclei in a hybird quark model approach: (i) hypernuclei with the configuration X–N–Λ, where X,N and Λ denote a nuclear core, nucleon (neutron or proton) and a hyperon respectively and (ii) hypernuclei with configuration closed core+Λ-hyperon. It is well known that the baryonic properties change inside a nuclear medium, thus the magnetic moment of Λ-hyperon well inside a six quark bag of ΛN would be different than that of the free hyperon. This can be a source of deviation of the magnetic moment values from the Schmidt limits.

In Chapter 2 we describe the hybrid quark model and different methods for evaluating the six-quark probability. Chapter 3 deals with the microscopic evaluation of the quark effects in the binding energy difference of mirror hypernuclei. Chapter 4 covers the quark effects in the magnetic moments of mirror nuclei and hypernuclei. The results obtained for the quark effects in the binding energy and magnetic moments are presented and discussed in Chapter 5 and Chapter 6 respectively.

2. HYBRID QUARK MODEL AND SIX-QUARK PROBABILITY

2

HYBRID QUARK MODEL AND SIX-QUARK PROBABILITY

2.1 Hybrid Quark Model

The correct theory which describes the quark degrees of freedom in nucleons explicitly is quantum chromodynamics (QCD) (*Wilczek* , 1982 and *Söding and Wolf* , 1981), but this is very difficult. Despite great progress in its perturbative (*Brown and Ellis*, 1981) and nonperturbative (*Wilson* , 1974; *Fucito et al*, 1982; *Creutz*, 1980 and *Shifman et al,* 1979) aspects there are only two basic pieces of information available, First the quarks have the property of asymptotic freedom, that is when they are close together they donot interact, second quarks are confined, which means when the separation distance between quarks is large the interaction between them is strong and attractive. This means quark, gluon and any object carrying color are confined. Thus if one nucleon carrying quark, gluon and pions is to interact with another nucleon that is well separated from it, all terms save the exchanges of color singlets are prohibited by the confinement property. Since the pion is the lighest color singlet object formed from a quark and antiquark pair, one expects the exchange of single pion to dominate the long range NN interaction. One also expects considerable contribution from the exchange of two pions. Thus at large separation distances the conventional picture of mesonic exchanges is in agreement with the QCD, but for small separation between nucleons the quarks in one nucleon overlap with the quarks in the other nucleon and this should be

treated as a system of confined quarks interacting via various perturbative terms of QCD.

In order to bridge the gap between the short distance perturbative QCD region and the long range pion exchange force *Henley et al* (1983) developed a hybrid quark model which retains the conventional meson exchange picture at long distances and represents the effects of QCD at short distances. This model is based on the coordinate space representation of nuclear systems. There are external regions in which separated baryons are representated as color singlets interaction via forces arising from the exchange of color singlet objects like pions. In the internal regions the quarks associated with two or more baryons interact with full color freedom.

Thus in hybrid model the nuclear matter has two phases. The nucleons are assumed to maintain their identity and properties as long as the distance between them is greater than a certain critical radius r_0 and if the distance between two nucleons is less than r_0 the system is treated as six-quarks. Symbolically the NN system can be represented as (*Greben and Thomas*, 1984)

$$\Psi = \mathcal{A}\ \Psi_1\ \Psi_2\ \phi_{12}\ (\boldsymbol{r}), \quad r > r_0 \tag{2.1-1}$$

$$\Psi = C\ \phi_6\ (\boldsymbol{r}_1, \ldots\ldots, \boldsymbol{r}_6), \quad r < r_0 \tag{2.1-2}$$

Here $\phi_{12}(\boldsymbol{r})$ is a conventional wavefunction obtained from solving the appropriate two nucleon Schrodinger equation. Ψ_1 and Ψ_2 represent the internal wavefunction of two nucleons, $\boldsymbol{r}_i$ is the coordinate of a single quark. We may define a six-body operator, $\boldsymbol{r}$, to represent the relative distance operator,

$$\boldsymbol{r} = \frac{\boldsymbol{r}_1 + \boldsymbol{r}_2 + \boldsymbol{r}_3}{3} - \frac{\boldsymbol{r}_4 + \boldsymbol{r}_5 + \boldsymbol{r}_6}{3} \tag{2.1-3}$$

The wave function for the six-quark system, can be represented in a simplified notation as,

$$\phi_6(\boldsymbol{r}_1.....\boldsymbol{r}_6) = \phi_6\ (\boldsymbol{r},\ \xi\ ) \tag{2.2-4}$$

in which ξ represents the other internal variables. The six-quark probability can be defined as (*Greben and Thomas* , 1984)

$$P_{6q} = |C|^2 \int \phi_6\ (\boldsymbol{r}_1.....\boldsymbol{r}_6)\, d\,\boldsymbol{r}_1.......d\,\boldsymbol{r}_6 \tag{2.1-5}$$

$$= |C|^2 \int |\phi_6(\boldsymbol{r},\xi)|^2 d^3r\, d\xi \tag{2.1-6}$$

Exact calculation of P_{6q} , within the constrains of QCD is difficult to make in a model independent way, but P_{6q} can be related to the external NN wavefunctions under different approximations.

The conservation of probability current across the boundary of matching radius at $r = r_0$ demands

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot j \tag{2.1-7}$$

$$\boldsymbol{r}\cdot\boldsymbol{j}\,\Big|_{r=r_0-\varepsilon} = \boldsymbol{r}\cdot\boldsymbol{j}\,\Big|_{r=r_0+\varepsilon} \tag{2.1-8}$$

Integrating the current conservation equation (2.1-7) over ξ and $\boldsymbol{r}$ with $r < r_0$,

$$\int d\xi\ d^3r\ \theta(r_0 - r)\,\frac{\partial \rho}{dt} = -\int d\xi\ d^3r\ \theta(r_0 - r)\ \nabla\cdot\boldsymbol{j} \tag{2.1-9}$$

Except for the time derivative, the left-hand side resembles P_{6q}. In the right-hand side the volume integral can be replaced by surface integral by the use of divergence theorem, so that the right-hand side of equation (2.1-9) depends on $\boldsymbol{r}\cdot\boldsymbol{j}$. Now using the equation of continuity of current equation

(2.1-8) the right-hand side of equation (2.1-9) can be expressed in terms of $\phi_{12}(\boldsymbol{r})$. Thus a very difficult expression involving six-quarks can be expressed in terms of ordinary nucleonic wavefunction evaluated at the boundary $r = r_0$, using the probability current conservation and the equation of continuity.

If the six-quark plus NN wavefunction obey the same normalization condition as an ordinary NN wavefunctions, then the six-quark probability equals the probability defect of $\phi_{12}(\boldsymbol{r})$ for $r < r_0$. This is the simplest prescription for the six-quark probability. But care has to be excercised in choosing the wavefunction $\phi_{12}(\boldsymbol{r})$. First, because of the different strong dynamics for $r < r_0$, the probability to find six-quarks with $r < r_0$ does not have to be the same as that of finding two nucleons with $r < r_0$ in the conventional picture. This change can be accommodated by allowing for a different normalization of the external wavefunction, even though its shape remains the same. Alternatively, the effective potential for $r > r_0$ may have to be modified to accommodate the different dynamics for $r < r_0$. This would lead to a different shape of the external wavefunction. Earlier calculation in nonrelativistic quark model framework indicate that there is no sudden decrease in the six-quark probability for small r. Sign change of the s -wave phase shift, which is usually explained by short range repulsion or equivalently by the vanishing of short-range NN wave function, can then be interpreted as the absence of NN components in the short-range six-quark wave function or as a node in the conventional wave function for small r. Thus if the short distance behaviour of NN potential is not represented by strong repulsion interaction, then the six-quark probability can be determined as a wavefunction defect of uncorrelated shell model wavefunction. Earlier studies on the continuum (*Henley et al*, 1983) and the bound state (*Miller*, 1984) wavefunction in the two body system show that

the current conservation guarantees identity of the six-quark probability and the conventional wavefunction defect for $r < r_0$ as long as we do not change the interaction for $r > r_0$. This is also true for the wave functions obtained with phenominal NN potential with modest short range repulsion.

2.2 Six-Quark Probability

The hybrid quark model can be generalized to a nucleon-nucleon pair inside a nucleus or a hypernucleus and the six-quark probability can be calculated from the shell model wave functions. Consider a hypernucleus consisting of A+1 nucleons and a hyperon. The ground state can be written as

$$\Psi^0(1,2\ldots\ldots A+1,\Lambda) = \phi_0^\Lambda \Psi_0^N$$
$$= \phi_{\alpha_0}(\Lambda)\mathcal{A}\left\{\prod_{i=1}^{A+1}\phi_{\alpha_i}(i)\right\} \tag{2.2-1}$$

where $\phi_{\alpha_i}(i)$ are normalized single particle states with quantum numbers α_i, $\phi_{\alpha_0}(\Lambda)$ is the hyperon state and $\mathcal{A}$ is the antisymmetrization operator. Since we are interested mostly with the state of the valence particle to form six quark bag with the core particles only, we define the wave function

$$\Psi_N^\upsilon(1,2\ldots\ldots A+1,\Lambda) = \phi_{\alpha_0}(\Lambda)\ \mathcal{A}\left\{\prod_{\alpha_i<\alpha_\upsilon}\left[1-\theta\left(r_0 - r_{\alpha_i\alpha_\upsilon}\right)\right]\prod_{i=1}^{A+1}\ \phi_{\alpha_i}(i)\right\} \tag{2.2-2}$$

Ψ_N^υ is written to ascertain that the valence particle in quantum state α_υ does not form a six-quark bag with any of the nucleons. The radius $r_{\alpha_i\alpha_\upsilon}$ should be considered as an operator defined by the following

$$r_{\alpha_i\alpha_\upsilon}\phi_{\alpha_i}(m)\ \phi_{\alpha_\upsilon}(n) = r_{mn}\phi_{\alpha_i}(m)\phi_{\alpha_\upsilon}(n). \tag{2.2-3}$$

Now by this notation $\mathcal{A}$ can operate directly on the single-particle wave function as it commutes with $r_{\alpha_i\alpha_\upsilon}$. The correlation function $\theta(r_0 - r_{mn})$ satisfies,

$$\theta\left(r_0 - r_{mn}\right) = 0 \text{ for } r_{mn} > r_0$$

$$= 1 \text{ otherwise}$$

Thus the probability of the valence particle being part of one or more six-quark bags with the core nucleons is

$$P_Q^{6q} = \left\langle \Psi^0 \middle| \Psi^0 \right\rangle - \left\langle \Psi_N^v \middle| \Psi_N^v \right\rangle \tag{2.2-4}$$

The valence nucleon can overlap with more than one core nucleons simultaneously and in addition to six quark bag can form nine-quark bag, twelve-quark bag, etc. Thus P_Q^{6q} can be broken up as

$$P_Q^{6q} = P_{Q_1}^{6q} + P_{Q_2}^{6q} + \ldots\ldots\ldots \tag{2.2-5}$$

where $P_{Q_1}^{6q}$ is the probability that the valence nucleon forms a six-quark bag with only one core nucleon, $P_{Q_2}^{6q}$ refers to the probability that valence nucleon forms a nine-quark bag with any two of the core nucleons and so on, (*Greben and Thomas* ,1984). Exact calculation of $P_{Q_1}^{6q}$, $P_{Q_2}^{6q}$ etc, is possible only for three body case. For heavier nuclei, if only the lowest order terms in the expansion of correlation function $\theta\left(r_0 - r_{ij}\right)$are retained, one can define an average probability $P_{NN}^{6q}\left(r_0\right)$ as a sum of single particle terms as,

$$P_{NN}^{6q}\left(r_0\right) = \sum_{\alpha_i = \alpha_1}^{\alpha_A} P_{\alpha_i}\left(r_0\right)$$

$$= \sum_{n_i l_i j_i \tau_{z_i}} \left(2 j_i + 1\right) P_{n_i l_i j_i \tau_{z_i}}\left(r_0\right) \tag{2.2-6}$$

The factor $\left(2 j_i + 1\right)$ arises on summing over the magnetic substates and

$$P_{\alpha_i}\left(r_0\right) = \left\langle \phi_{\alpha_v}(1)\, \phi_{\alpha_i}(2) \middle| \theta\left(r_0 - r_{12}\right) \middle| \phi_{\alpha_v}(1)\, \phi_{\alpha_i}(2) - \phi_{\alpha_i}(1) \phi_{\alpha_v}(2) \right\rangle, \tag{2.2-7}$$

where $\alpha_v(=n_v l_v j_v)$ and $\alpha_i(=n_i l_i j_i)$ define the quantum states of the valence and the core nucleons respectively. $P_{n_i l_i j_i \tau_{z_i}}(r_0)$ can be interpreted as the probability for the valence particle to be within a distance r_0 of a specified core particle with quantum numbers $n_i l_i j_i \tau_{z_i}$ and is

$$P_{n_i l_i j_i \tau_{z_i}}(r_0) = \frac{1}{(2j_v+1)(2j_i+1)} \times \sum_{m_v m_i} \left\langle \phi_{\alpha_v}(1)\phi_{\alpha_i}(2) \middle| \theta(r_0 - r_{12}) \middle| \phi_{\alpha_v}(1)\phi_{\alpha_i}(2) - \phi_{\alpha_i}(1)\,\phi_{\alpha_v}(2) \right\rangle \tag{2.2-8}$$

If we assume that the difference between neutron and proton orbits can be ignored, the isospin index can be suppressed. From equations (2.2-6) and (2.2-8), $P_{NN}^{6q}(r_0)$can be expressed as a combination of a direct term $P_{n_i l_i j_i}^{d}(r_0)$and an exchange term $P_{n_i l_i j_i}^{e}(r_0)$as,

$$P_{NN}^{6q}(r_0) = \sum_{n_i l_i j_i}(2j_i+1)\left[2P_{n_i l_i j_i}^{d}(r_0) - P_{n_i l_i j_i}^{e}(r_0)\right] \tag{2.2-9}$$

The factor 2 in $P_{n_i l_i j_i}^{d}(r_0)$ stems from identical proton and neutron direct contribution.The probability $P_{Q_1}^{6q}(r_0)$, that the valence particle forms a six-quark bag with only one core nucleon is

$$P_{Q_1}^{6q}(r_0) = \left\langle \Psi_{Q_1}^{v} \middle| \Psi_{Q_1}^{v} \right\rangle \tag{2.2-10}$$

where

$$\left| \Psi_{Q_1}^{v} \right\rangle = \phi_{\alpha_0}(A)\mathcal{A}\left\{ \sum_{\alpha_i} \theta\left(r_0 - r_{\alpha_i \alpha_v}\right) \prod_{\alpha_j \neq \alpha_i}^{\alpha_A} \left[1 - \theta\left(r_0 - r_{\alpha_j \alpha_v}\right)\right] \prod_{i=1}^{A+1} \phi_{\alpha_i}(i) \right\} \tag{2.2-11}$$

Similarly the probability $P_{Q_2}^{6q}(r_0)$, that the valence particle forms a nine-quark bag with any two of the core nucleons is

$$P_{Q_2}^{6q}(r_0) = \left\langle \Psi_{Q_2}^{v} \middle| \Psi_{Q_2}^{v} \right\rangle \tag{2.2-12}$$

where

$$\left| \Psi_{Q_2}^{v} \right\rangle = \phi_{\alpha_0}(\Lambda)\mathcal{A} \left\{ \sum_{\alpha_i \alpha_j} \theta\left(r_0 - r_{\alpha_i \alpha_v}\right) \theta\left(r_0 - r_{\alpha_j \alpha_v}\right) \prod_{\alpha_m \neq \alpha_j \neq \alpha_i}^{\alpha_A} \left[1 - \theta\left(r_0 - r_{\alpha_m \alpha_v}\right)\right] \right.$$

$$\left. \times \prod_{i=1}^{A+1} \phi_{\alpha_i}(i) \right\} \tag{2.2-13}$$

The quadratic higher order terms in the correlation function in $P_{Q_1}^{6q}(r_0)$ and $P_{Q_2}^{6q}(r_0)$ can be reduced to first order by using the following identity,

$$\theta\left(r_0 - r_{ij}\right) \theta\left(r_0 - r_{ij}\right) = \theta\left(r_0 - r_{ij}\right). \tag{2.2-14}$$

Even then calculation of $P_{Q_1}^{6q}(r_0)$ and $P_{Q_2}^{6q}(r_0)$ etc, for heavier nuclei becomes more and more difficult. If we assume that the chance for the valence particle to overlap with the core particle does not depends upon whether it already overlaps with other core particles, one can calculate $P_{Q_1}^{6q}(r_0)$ and $P_{Q_2}^{6q}(r_0)$ from the average probability $P_{NN}^{6q}(r_0)$ using the following expressions,

$$P_{Q_1}^{6q}(r_0) = P_{NN}^{6q}(r_0)\left(1 - P_{NN}^{6q}(r_0)/A\right)^{A-1} \tag{2.2-15}$$

and

$$P_{Q_2}^{6q}(r_0) = \left[\frac{A}{2}\right] \left[\frac{P_{NN}^{6q}(r_0)}{A}\right]^2 \left[1 - \frac{P_{NN}^{6q}(r_0)}{A}\right]^{A-2} \tag{2.2-16}$$

The valence nucleon can also overlap with the hyperon and from a six-quark bag with the hyperon. The corresponding overlap probability can be expressed as,

$$P_{\Lambda N}^{6q}(r_0) = P'_{n_0 l_0 j_0}(r_0) \tag{2.2-17}$$

with

$$P'_{n_0 l_0 j_0}(r_0) = \frac{1}{(2j_v+1)(2j_0+1)}$$

$$\times \sum_{m_0 m_v} \left\langle \phi_{\alpha_0}(1)\, \phi_{\alpha_v}(2) \middle| \theta(r_0 - r_{12}) \middle| \phi_{\alpha_0}(1)\, \phi_{\alpha_v}(2) \right\rangle \tag{2.2-18}$$

There is no exchange term for ΛN overlapping. The overlap probability $P_{NN}^{6q}(r_0)$ and $P_{\Lambda N}^{6q}(r_0)$ as defined in equations (2.2-8), (2.2-9) and (2.2-18) respectively, can be estimated either by Moshinsky transformation method or by Slater integral method.

2.2a Moshinsky Transformation Method

When the nucleons or hyperon in nuclei are described by harmonic oscillator wavefunctions the matrix elements in equations (2.2-8), (2.2-18) and (2.2-9) can be evaluated by Moshinsky transformation method described in Appendix A. The matrix elements in equation (2.2-8) are transformed to relative and centre-of-mass basis, the transformation coefficients being Moshinsky brackets (*Moshinsky*, 1959 and *Brody,* 1960). The final expression for the direct term in equation (2.2-9) simplifies to

$$P_{n_i l_i j_i}^{d}(r_0) = \sum_{\substack{\lambda SnlNL \\ JM}} A^2 \begin{bmatrix} l_i & ½ & j_i \\ l_v & ½ & j_v \\ \lambda & S & J \end{bmatrix}$$

$$\times \langle nlNL; \lambda | n_i l_i n_v l_v; \lambda \rangle_{MB}^2 \langle nl \| \theta(r_0 - r) \| nl \rangle \tag{2.2a-1}$$

where

$$\langle nl \| \theta(r_0 - r) \| nl \rangle = \int_0^{r_0} R_{nl}^2(r)\, dr,$$

$R_{nl}(r)$ are normalized radial functions. The quantum numbers $nlSJ$ and NL refer to the relative and centre-of-mass (CM) state of the overlapping pair respectively. The angular momentum J is the result of coupling l and S, and λ is the result of coupling l and L . $\langle nlNL;\lambda|n_il_in_vl_v;\lambda\rangle_{MB}$ are the Moshinsky brackets. The expression for the exchange term is similar to equation (2.2a-1) with an additional factor of $(-1)^{\lambda+S+l_i+l_v+1}$. In the calculation of $P^{6q}_{\Lambda N}(r_0)$, to facilitate Moshinsky transformation to relative basis in the matrix elements in equation (2.2-18), we have chosen $\nu_\Lambda=(m_\Lambda/m_N)\,\nu_N$. With this ansatz these matrix elements can be expanded in relative and center-of-mass basis, the expansion coefficients being Smirnov coefficients. Thus equation (2.2-18) can also be simplified to an expression similar to equation (2.2a-1) with Moshinsky brackets replaced by Smirnov brackets $\langle nlNL;\lambda|n_0l_0n_vl_v;\lambda\rangle_{SM}$ (*Smirnov*, 1961, 1962).

2.2b Slater Integral Method

The function $\theta(r_0-r_{12})$ can be expanded in complete set of Legendre polynomials of the angle θ between the vectors $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$, where $\boldsymbol{r}=\boldsymbol{r}_1-\boldsymbol{r}_2$. Thus,

$$\theta(r_0-r_{12})=\sum_{\lambda=0}^{\infty}f_\lambda(r_1,r_2)P_\lambda(\cos\theta) \tag{2.2b-1}$$

The unknown quantities $f_\lambda(r_1,r_2)$ are given in terms of $\theta(r_0-r_{12})$ by the integral

$$f_\lambda(r_1,r_2)=\tfrac{2\lambda+1}{2}\int_0^\infty\theta(r_0-r_{12})P_\lambda(\cos\theta)\,d(\cos\theta) \tag{2.2b-2}$$

Using equations (2.2b-1) and (2.2b-2) in the matrix elements of equation (2.2-8) the direct and the exchange terms reduce to the following expression after some standard angular momentum algebra,

$$P^d_{n_il_ij_i}(r_0)=\tfrac{1}{2}\int_0^\infty dr_1r_1^2\int_0^\infty dr_2r_2^2\left|\phi_{n_il_ij_i}(r_1)\,\phi_{n_vl_vj_v}(r_2)\right|^2\int_{-1}^1 d(\cos\theta)\theta(r_0-r_{12}) \tag{2.2b-3}$$

and

$$P^{e}_{n_i l_i j_i}(r_0) = \int_0^\infty dr_1 r_1^2 \int_0^\infty dr_2 r_2^2 \; e_{l_i j_i l_v j_v}(r_1, r_2)\; \phi^*_{n_i l_i j_i}(r_1)\phi^*_{n_i l_i j_i}(r_2)\phi_{n_v l_v j_v}(r_1')\phi_{n_v l_v j_v}(r_2) \tag{2.2b-4}$$

where

$$e_{l_i j_i l_v j_v}(r_1, r_2) = (2l_i+1)(2l_v+1)\sum_\lambda (2\lambda+1)\left[\begin{pmatrix} l_i & l_v & \lambda \\ 0 & 0 & 0 \end{pmatrix}\begin{Bmatrix} \lambda & l_i & l_v \\ ½ & j_v & j_i \end{Bmatrix}\right]^2$$

$$\times \tfrac{1}{2}\int_{-1}^{1} d(\cos\theta)\,\theta(r_0 - r_{12})P_\lambda(\cos\theta) \tag{2.2b-5}$$

where $\phi_{n_i l_i j_i}(r_1)\;\phi_{n_v l_v j_v}(r_2)$ in equation (2.2b-3) are the radial wave functions for core nucleon and valence nucleon respectively. Finally $P^{6q}_{NN}(r_0)$ is calculated using equations (2.2-9), (2.2b-3) and (2.2b-4). For $P^{6q}_{AN}(r_0)$ only the direct term contributes and there is no exchange term. The detailed method of calculating these integrals for harmonic oscillator wavefunctions is described in Appendix B.

3. QUARK EFFECTS IN BINDING ENERGY DIFFERENCE OF MIRROR HYPERNUCLEI

3

QUARK EFFECTS IN BINDING ENREGY DIFFERENCE OF MIRROR HYPERNUCLEI

3.1 Binding Energy of Mirror Hypernuclei

The pair of mirror hypernuclei ${}^{A+2}_{\Lambda}X$ and ${}^{A+2}_{\Lambda}Y$ are formed by adding a hyperon and a proton or a neutron, to the core nuclei consisting of Z protons and A nucleons as $M_c(A,Z)$. The mass of these hypernuclei pair can be defined as,

$$M\left({}^{A+2}_{\Lambda}X\right)=M_c(A,Z)+m_\Lambda+m_n-B.E.\left({}^{A+2}_{\Lambda}X\right) \tag{3.1-1}$$

$$M\left({}^{A+2}_{\Lambda}Y\right)=M_c(A,Z)+m_\Lambda+m_p-B.E.\left({}^{A+2}_{\Lambda}Y\right) \tag{3.1-2}$$

If we neglect the core contribution which is irrelevent for the binding energy difference and consider only the effect of six-quark cluster formation of the valence nucleon with the core nucleon and the hyperon, an additional contribution to the binding energies of the hypernuclei results. If the probabilities that the valence nucleon forms a six-quark bag with any of the core nucleons and hyperon are $P^{6q}_{NN}(r_0)$and $P^{6q}_{\Lambda N}(r_0)$ respectively and P_0 is the probability that it does not overlap with any of the nucleons or hyperon then,

$$P_0 + P_{NN}^{6q}(r_0) + P_{\Lambda N}^{6q}(r_0) \cong 1 \tag{3.1-3}$$

Assuming that the core proton and core neutron probabilities are identical (a reasonable assumption considering that we deal with $N = Z$ cores), equations (3.1-1) and (3.1-2) can be rewritten using equation (3.1-3) as,

$$\begin{aligned} M'\left({}^{A+2}_{\Lambda}Y\right) &= M_c(A,Z)+P_0 m_p+\frac{1}{2}P_{NN}^{6q}(r_0)\,(m_{pp}-m_p)+\frac{1}{2}P_{NN}^{6q}(r_0)\,(m_{pn}-m_n) \\ &+ m_\Lambda + P_{\Lambda N}^{6q}(r_0)(m_{p\Lambda}-m_\Lambda)-\left\{B.E.\left({}^{A+2}_{\Lambda}Y\right)+\Delta V_c\left({}^{A+2}_{\Lambda}Y\right)\right\} \end{aligned} \tag{3.1-4}$$

where m_{pp}, m_{pn} and $m_{p\Lambda}$ represent the masses of six-quark bags formed of two protons, a proton and a neutron, and a proton and a hyperon respectively. $\Delta V_c\left({}^{A+2}_{\Lambda}Y\right)$ measures how much Coulomb repulsion is lost by cutting the two proton integral off at distances $r < r_0$. Due to this the binding energy in ${}^{A+2}_{\Lambda}Y$ increases by $\Delta V_c\left({}^{A+2}_{\Lambda}Y\right)$. We may rewrite equation (3.1-4) as,

$$M'\left({}^{A+2}_{\Lambda}Y\right) = M_c(A,Z)+m_\Lambda+m_p-\left\{B.E.\left({}^{A+2}_{\Lambda}Y\right)+\delta B\left({}^{A+2}_{\Lambda}Y\right)\right\} \tag{3.1-5}$$

where

$$\begin{aligned} \delta B\left({}^{A+2}_{\Lambda}Y\right) &= P_{NN}^{6q}(r_0)\,m_p + P_{\Lambda N}^{6q}(r_0)\,m_p - \frac{1}{2}P_{NN}^{6q}(r_0)\left(m_{pp}-m_p\right) \\ &- \frac{1}{2}P_{NN}^{6q}(r_0)\left(m_{pn}-m_n\right)-\frac{1}{2}P_{\Lambda N}^{6q}(r_0)\left(m_{p\Lambda}-m_\Lambda\right) \\ &+ \Delta V_c\left({}^{A+2}_{\Lambda}Y\right). \end{aligned} \tag{3.1-6}$$

Similarly

$$\begin{aligned} \delta B\left({}^{A+2}_{\Lambda}X\right) &= P_{NN}^{6q}(r_0)m_n + P_{\Lambda N}^{6q}(r_0)m_n - \frac{1}{2}P_{NN}^{6q}(r_0)(m_{np}-m_n) \\ &- \frac{1}{2}P_{NN}^{6q}(r_0)\left(m_{nn}-m_n\right)-\frac{1}{2}P_{\Lambda N}^{6q}(r_0)\left(m_{n\Lambda}-m_\Lambda\right), \end{aligned} \tag{3.1-7}$$

where m_{nn}, m_{np} and $m_{n\Lambda}$ are the masses of six-quark bags of two neutrons, a neutron and a proton, and a neutron and a hyperon respectively (*Köch and Miller*, 1985). Comparision of equation (3.1-5) with equation (3.1-2) shows that the six-quark cluster formation of the valence nucleon with the core nucleons and the hyperon increases the binding energy of Λ–particle in ${}^{A+2}_{\Lambda}Y$ by $\delta B\left({}^{A+2}_{\Lambda}Y\right)$. The equations (3.1-6) and (3.1-7) give the additional contribution to the binding energy for ${}^{A+2}_{\Lambda}Y$ and ${}^{A+2}_{\Lambda}X$ respectively. Concentrating on the effect of six-quarks cluster formation only on the binding energy difference (denoted by ΔB_{6q}), we get

$$\begin{aligned}\Delta B_{6q} &= \left[\delta B\left({}^{A+2}_{\Lambda}X\right)-\delta B\left({}^{A+2}_{\Lambda}Y\right)\right]\\ &=\frac{1}{2}P^{6q}_{NN}\left(r_0\right)\left(2m_n-2m_p\right)-\frac{1}{2}P^{6q}_{NN}\left(r_0\right)\left(m_{nn}-m_{pp}\right)\\ &+P^{6q}_{\Lambda N}\left(r_0\right)\left(m_n-m_p\right)-P^{6q}_{\Lambda N}\left(r_0\right)\left(m_{n\Lambda}-m_{p\Lambda}\right)-\Delta V_c\left({}^{A+2}_{\Lambda}Y\right)\end{aligned} \tag{3.1-8}$$

If instead of the average probability $P^{6q}_{NN}(r_0)$, we use the valence probabilities $P_0, P^{6q}_{Q_1}(r_0), P^{6q}_{Q_2}(r_0)$ and $P^{6q}_{\Lambda N}(r_0)$, the completness of the wave function demands,

$$P_0+P^{6q}_{Q_1}\left(r_0\right)+P^{6q}_{Q_2}\left(r_0\right)+P^{6q}_{\Lambda N}\left(r_0\right)\cong 1 \tag{3.1-9}$$

If terms of order higher than $P^{6q}_{Q_2}\left(r_0\right)$ are neglected, equation (3.1-4) gets modified to

$$\begin{aligned}M\left({}^{A+2}_{\Lambda}Y\right) &= M_c\left(A,Z\right)+P_0\,m_p+\frac{1}{2}P^{6q}_{Q_1}\left(r_0\right)\left(m_{pp}-m_p\right)\\ &+\frac{1}{2}P^{6q}_{Q_1}\left(r_0\right)\left(m_{pn}-m_n\right)+\frac{1}{2}P^{6q}_{Q_2}\left(r_0\right)\left(m_{ppn}-m_p-m_n\right)\\ &+\frac{1}{4}P^{6q}_{Q_2}\left(r_0\right)\left(m_{ppp}-2m_p\right)+\frac{1}{4}P^{6q}_{Q_2}\left(r_0\right)\left(m_{pnn}-2m_n\right)\\ &+m_\Lambda+P^{6q}_{\Lambda N}\left(r_0\right)\left(m_{p\Lambda}-m_\Lambda\right)-\left\{B.E.\left({}^{A+2}_{\Lambda}Y\right)+\Delta V_c\left({}^{A+2}_{\Lambda}Y\right)\right\}\end{aligned} \tag{3.1-10}$$

similarly

$$M\left({}^{A+2}_{\Lambda}X\right) = M_c(A,Z) + P_0 m_n + \frac{1}{2} P^{6q}_{Q_1}(r_0)\left(m_{nn} - m_n\right)$$

$$+\frac{1}{2} P^{6q}_{Q_1}(r_0)\left(m_{pn} - m_p\right) + \frac{1}{2} P^{6q}_{Q_2}(r_0)\left(m_{nnp} - m_p - m_n\right)$$

$$+\frac{1}{4} P^{6q}_{Q_2}(r_0)\left(m_{nnn} - 2m_n\right) + \frac{1}{4} P^{6q}_{Q_2}(r_0)\left(m_{npp} - 2m_p\right)$$

$$+m_\Lambda + P^{6q}_{\Lambda N}(r_0)\left(m_{n\Lambda} - m_\Lambda\right) - \left\{B.E.\left({}^{A+2}_{\Lambda}X\right) + \Delta V_c\left({}^{A+2}_{\Lambda}X\right)\right\} \quad (3.1\text{-}11)$$

where m_{ppp} and m_{nnn} are the masses of nine-quark bags formed of three protons and three neutrons. Similarly $m_{pnn}, m_{ppn}, m_{nnp}$ and m_{npp} represent the masses of nine-quark bags formed of one proton-two neutrons, two protons-one neutron, two neutrons-one proton, and one neutron-two protons respectively. Finally from equations (3.1-10) and (3.1-11) following the same treatment, the binding energy difference comes out to be,

$$\Delta B'_{6q} = \frac{1}{2} P^{6q}_{Q_1}(r_0)\left(m_{pp} - m_{nn} + 2m_n - 2m_p\right)$$

$$+\frac{1}{4} P^{6q}_{Q_2}(r_0)\left(m_{ppp} - m_{nnn} + m_{ppn} - m_{nnp} + 4m_n - 4m_p\right)$$

$$+P^{6q}_{\Lambda N}(r_0)\left(m_{p\Lambda} - m_{n\Lambda} + m_n - m_p\right) - \Delta V_c\left({}^{A+2}_{\Lambda}Y\right) \quad (3.1\text{-}12)$$

The equation (3.1-8) gives the binding energy difference $\left(\Delta B_{6q}\right)$ in terms of $P^{6q}_{NN}(r_0)$ and $P^{6q}_{\Lambda N}(r_0)$ whereas $\Delta B'_{6q}$ in equation (3.1-12) gives the binding energy difference in terms of $P^{6q}_{Q_1}(r_0), P^{6q}_{Q_2}(r_0)$ and $P^{6q}_{\Lambda N}(r_0)$ respectively.

3.2 Mass Difference of Six-Quark bags of Neutrons and Protrons

The mass difference of six-quark bags formed from two protons and two neutrons $\left(m_{pp} - m_{nn}\right)$ can be computed either in the nonrelativistic quark model (NRQM) (*Rujula et al* , 1975; *Isgur and Karl,* 1978 and *Isgur,* 1980) or in the MIT bag model (*Köch and Miller* , 1985).

3.2a Nonrelativistic Quark Model

In NRQM following *Rujula et al* (1975) the mass of the n-quark system is given by

$$M = M_o + \sum_{i=1}^{n}\left[\left(m_i - \overline{m}\right) + a_n\left[\frac{1}{m_i} - \frac{1}{\overline{m}}\right]\right] + \sum_{i<j}\left(\alpha Q_i Q_j + \lambda_i \cdot \lambda_j \ \alpha_s\right)$$

$$\left[b_n - \frac{c_n}{m_i m_j} - d_n\left[\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{16}{3}\ \frac{s_i \cdot s_j}{m_i m_j}\right]\right] \qquad (3.2a\text{-}1)$$

where M_0 is the mass of the unperturbed system obtained when there is no interaction between quarks; s_i and m_i are the spin and masses of the quarks; $Q_i Q_j$ is the product of quark charge factors; $\lambda_i \cdot \lambda_j$ is the dot product of the two Gell-Mann SU(3) matrices; α and α_s are the fine structure constant and strong interaction constant respectively. The terms a_n, b_n, c_n and d_n are expectation values of various operators in the antisymmetrized unperturbed n-quark wave function Ψ_n. For three quark system the spatial wave function is a product of harmonic oscillator wave functions (*Close*, 1979).

$$\Psi_3(\rho, \lambda) = Ne^{-\left(\rho^2 + \lambda^2\right)/\left(2b^2\right)}, \qquad (3.2a\text{-}2)$$

where the three-body Jacobian coordinates

$$\rho = (\boldsymbol{r}_1 - \boldsymbol{r}_2)/\sqrt{2}$$

$$\lambda = (\boldsymbol{r}_1 + \boldsymbol{r}_2 - 2\boldsymbol{r}_3)/\sqrt{6}, \qquad (3.2a\text{-}3)$$

are employed. Here $\boldsymbol{r}_1 + \boldsymbol{r}_2 + \boldsymbol{r}_3 = 0$ and the normalization constant is $N = 1/(\pi^{1/2}b)^3$.

Thus the constants for n = 3 quark system are

$$a_3 = \frac{1}{2}\frac{1}{b^2}\left[1 - \frac{15}{8b^2\overline{m}^2}\right],$$

$$b_3 = \left[\frac{2}{\pi}\right]^{1/2}\frac{1}{b},$$

$$c_3 = -\left[\frac{2}{\pi}\right]^{1/2}\frac{1}{3b^3},$$

$$d_3 = \frac{1}{8}\left[\frac{2}{\pi}\right]^{1/2}\frac{1}{b^3}. \qquad (3.2a\text{-}4)$$

From equation (3.2a-1) the neutron and proton mass difference can be expressed as

$$M_n - M_p = \Delta m\left\{1 - \frac{a_3}{\overline{m}^2} - \frac{2}{3}(\alpha + 2\alpha_s)\frac{1}{\overline{m}^3}\left[(c_3 + 2d_3) + \frac{4}{3}d_3\right] - \frac{b_3\alpha}{3}\right\} \qquad (3.2a\text{-}5)$$

where

$$\overline{m}^2 = m_d\, m_u,$$

In equation (3.2a-5), by putting in the known experimental value of neutron-proton mass difference of 1.29MeV and fixing the values of constants b and

$\overline{m}$ the difference $\Delta m = m_d - m_u$ can be evaluated. This value can be fed in equation (3.2a-1) with $n = 6$ to calculate the six-quark mass difference $m_{nn} - m_{pp}$ as,

$$m_{nn} - m_{pp} = 2(m_d - m_u)\left\{1 - \frac{a_6}{\overline{m}^2} - \frac{2}{9\overline{m}^3}\left[-\frac{5}{2}\,\alpha\,(c_6 + 2d_6) + 2\alpha d_6\right]\right.$$
$$\left. - \frac{\alpha_s}{\overline{m}^3}\left[\frac{4}{3}(c_6 + 2d_6) + 5.33\, d_6\right]\right\}$$
$$- \frac{1}{3\overline{m}^2}\left[-5\alpha\,(c_6 + 2d_6) + 4\alpha d_6\right] - \frac{5}{3}\,\alpha b_6 \qquad (3.2\text{a-}6)$$

If the six-quark spatial wave function Ψ_6 is given by

$$\Psi_6 = N^{5/2}\, e^{-\sum_{i=2}^{6} \xi_i^2/(2b^2)} \qquad (3.2\text{a-}7)$$

where

$$\xi_i = \left[(i-1)\, r_i - \sum_{j=1}^{i-1} r_j\right]\Big/\sqrt{i(i-1)} \qquad (3.2\text{a-}8)$$

the resulting values of constants a_6, b_6, c_6 and d_6 are

$$a_6 = \frac{5}{8}\left(\frac{1}{b^2}\right)\left[1 - \frac{5}{8}\,\frac{1}{\overline{m}^2 b^2}\right],$$
$$b_6 = \left[\frac{2}{\pi}\right]^{1/2}\left(\frac{1}{b}\right),$$
$$c_6 = -\left[\frac{2}{\pi}\right]^{1/2}\left(\frac{1}{6b^3}\right),$$
$$d_6 = \left[\frac{2}{\pi}\right]^{1/2}\left(\frac{1}{8b^3}\right). \qquad (3.2\text{a-}9)$$

3.2b MIT Bag Model

In the MIT bag model QCD relativistic quarks are confined within a spherical bag. Confinement is imposed by requiring that no color flux passes through the surface and that the outward pressure caused by quarks colliding with the inner surface is balanced by the inward pressure of the QCD vacuum which surrounds the bag. The Hamiltonian consists of the following terms : quark energy, magnetic one-gluon exchange interaction energy and electromagnetic interaction. The relevent terms (*Köch and Miller*, 1985) which contribute to the mass difference of neutron and proton bags are :

(i) Quark Energy (E_k)

$$E_k = \sum_{\substack{a \\ flavors}} n_a w(m_a R)/R, \tag{3.2b-1}$$

where w is the eigen frequency implied by the non-linear boundary condition.

(ii) Color hyperfine interaction (E_m)

$$E_m = \frac{\alpha_c}{R} \sum_{\substack{a,b \\ flavors}} M(m_a R, m_b R) \sum_{\substack{i<j \\ n}} \left\langle \sigma_i \cdot \sigma_j \lambda_i^n \cdot \lambda_j^n \right\rangle \tag{3.2b-2}$$

where R is the bag radius and n_a is the number of quarks of flavor a and mass m_a. The mode frequencies are $w(m_a R)$ and $M(m_a R, m_b R)$ is an interaction strength arising from one-gluon exchange.

(iii) Electromagnetic interaction E_{em}

$$E_{em} = \frac{1.5 MeV fm}{R} \sum_{i<j} Q_i Q_j, \tag{3.3b-3}$$

This includes both the Coulomb and magnetic interaction between the quark pairs. From equations (3.2b-1), (3.2b-2) and (3.2b-3) the neutron-proton mass difference can be expressed as

$$M_n - M_p = \left[\frac{dw(\overline{m}R_3)}{d\overline{m}R_3} + \frac{8}{3}\,\alpha_c\,\frac{dM(\overline{m}R_3, \overline{m}R_3)}{d(\overline{m}R_3)} \right] \Delta m \; - \frac{0.5 MeV\; fm}{R_3} \tag{3.2b-4}$$

where $\overline{m}$ is the average of up and down quark masses and R_3 is the three-quark bag radius. By *Bickerstaff and Thomas* (1982) numerical solution of the Dirac equation for a quark confined in a cavity of radius R_3 approximatly equal to $1fm$, the derivatives for the average mass $\overline{m}$ are,

$$\frac{dw}{d(\overline{m}R_3)} = 0.487$$

and

$$\frac{dM}{d(\overline{m}R_3)} = -\,0.047 \tag{3.2b-5}$$

By putting in the experimental values of neutron-proton mass difference of $1.29 MeV$ and the values of the derivatives from equation (3.2b-5) in equation (3.2b-4) the value of $\Delta m = m_d - m_u$ can be calculated. This in turn is used to compute the mass difference of six-quark bags formed from two neutron and two proton $(m_{nn} - m_{pp})$ using equations (3.2b-1), (3.2b-2) and (3.2b-3). The $m_{nn} - m_{pp}$ in the bag model is

$$m_{nn} - m_{pp} = \left[2.\frac{dw(\overline{m}R_6)}{d(\overline{m}R_6)} + 16\alpha_c\,\frac{dM(\overline{m}R_6, \overline{m}R_6)}{d(\overline{m}R_6)} \right] \Delta m \\ - \frac{2.5 MeV fm}{R_6}, \tag{3.2b-6}$$

where R_6 is the radius of the six-quark bag. From the bag virial theorem $M \propto R^3$ and thus

$$R_6 = 2^{1/3} R_3 \tag{3.2b-7}$$

Again the derivatives are obtained from the numerical solution of the Dirac equation for a quark confined in a cavity of radius R_6, for $R_6 \approx 1.2 fm$.

$$\frac{dw(\overline{m}R_6)}{d(\overline{m}R_6)} = 0.489$$

and

$$\frac{dM(\overline{m}R_6, \overline{m}R_6)}{d(\overline{m}R_6)} = 0.047 \tag{3.2b-8}$$

A simplified expression (*Greben and Thomas*, 1984) for the mass of $3n$-quark bag can be written as

$$E = 1.5 \sum_{i<j}^{3n} \frac{Q_i Q_j}{R_{3n}} + 0.42 \sum_{i=1}^{3n} \frac{c_i}{R_{3n}}, \tag{3.2b-9}$$

where hyperfine interaction terms is approximatly accounted for by using the value 0.42 instead of 0.49 in the quark energy term. The terms which do not contribute to the mass difference are omitted. Q_i and m_i are the charge and the mass of the i^{th} quark respectively and R_{3n} is the radius of $3n$-quark bag, where $m_i = c_i/R$.

3.3 Coulomb Energy

The Coulomb energy of hypernuclei can be estimated by evaluating the average value of interaction between all pairs of protons in the nucleus. In calculating the Coulomb energy difference between the mirror pair of hypernuclei, contribution from the core protons cancels out, being common to both the members of the pair. The difference in the Coulomb energy (V_c) of the mirror pair is only due to the Coulomb interaction of valence proton

in the proton rich partner with the core proton. This inturn can be expressed as the sum of two body matrix element of Coulomb interaction between the valence proton and the core protons in the quantum states α_v and α_i respectively (*Nolen and Schiffer*, 1969).

$$V_C = \sum_{\alpha_i} \Delta_{\alpha_v \alpha_i} \,, \qquad (3.3\text{-}1)$$

where $\Delta_{\alpha_v \alpha_i}$ is the antisymmetrized matrix element of the Coulomb interaction between the valence proton and the core proton and the sum is over quantum states of the core proton. For completely filled shells V_c can be expressed as

$$V_c = \sum_{n_i l_i j_i} (2j_i + 1)\left[\Delta_D - \Delta_{exch}\right] \qquad (3.3\text{-}2)$$

where

$$\Delta_D = \left\langle \phi_{\alpha_v}(1)\, \phi_{\alpha_i}(2) \left| \frac{e^2}{r_{12}} \right| \phi_{\alpha_v}(1)\, \phi_{\alpha_i}(2) \right\rangle$$

$$\Delta_{exch} = \left\langle \phi_{\alpha_v}(1)\, \phi_{\alpha_i}(2) \left| \frac{e^2}{r_{12}} \right| \phi_{\alpha_v}(2)\, \phi_{\alpha_i}(1) \right\rangle \qquad (3.3\text{-}3)$$

The e^2/r_{12} is the Coulomb operator for protons separated by a distance r_{12}. Δ_D and Δ_{exch} are the direct and exchange terms respectively. Equations (3.3-3) and (3.3-2) can be evaluated either by Slater integral method or by Moshinsky transformation method.

Slater Integral Method : In the matrix elements of equation (3.3-3) the direct and the exchange terms after integration over the angular variables can be written in terms of Slater integral as

$$\Delta_D = \frac{1}{2}\int_0^\infty dr_1\, r_1^2 \int_0^\infty dr_2\, r_2^2 \left|\phi_{n_i l_i j_i}(r_1)\, \phi_{n_v l_v j_v}(r_2)\right|^2 \int_{-1}^{1} \frac{e^2}{r_{12}}\, d(\cos\theta) \qquad (3.3\text{-}4)$$

and

$$\Delta_{exch} = \int_0^\infty dr_1\, r_1^2 \int_0^\infty dr_2\, r_2^2\, e'_{l_i j_i l_v j_v}(r_1,r_2)\, \phi^*_{n_i l_i j_i}(r_1)\, \phi^*_{n_i l_i j_i}(r_2)\, \phi_{n_v l_v j_v}(r_1)\, \phi_{n_v l_v j_v}(r_2)$$

, (3.3-5)

where

$$e'_{l_i j_i l_v j_v}(r_1,r_2) = (2l_i+1)(2l_v+1)\sum_\lambda (2\lambda+1)\left[\begin{pmatrix} l_i & l_v & \lambda \\ 0 & 0 & 0 \end{pmatrix}\begin{Bmatrix} \lambda & l_i & l_v \\ \frac{1}{2} & j_v & j_i \end{Bmatrix}\right]^2$$

$$\times \frac{1}{2}\int_{-1}^{1} \frac{e^2}{r_{12}} P_\lambda(\cos\theta)\, d(\cos\theta) \qquad (3.3\text{-}6)$$

The detailed method of calculating these integrals is described in Appendix B.

Moshinsky Transformation Method : The matrix element in equation (3.3-2) are transformed to relative and centre-of-mass coordinates and the expression for the direct term in equation (3.3-3) simplifies to

$$\Delta_D = \frac{1}{(2j_v+1)} \sum_{\substack{\lambda S n l N L \\ J M}} A^2 \begin{bmatrix} l_i & \frac{1}{2} & j_i \\ l_v & \frac{1}{2} & j_v \\ \lambda & S & J \end{bmatrix} \langle nlNL;\lambda | n_i l_i n_v l_v;\lambda\rangle^2_{MB} \left\langle nl \left\| \frac{e^2}{r} \right\| nl \right\rangle$$

(3.3-7)

The expression for the exchange term is similar to equation (3.3-7) with an additional phase factor of $(-1)^{\lambda+S+l_i+l_v+1}$. The loss in Coulomb energy ΔV_c when the two protons overlap for $r < r_0$ can be obtained from an expression similar to equation (3.3-7) with the radial matrix element equal to $\left\langle nl \left\| \theta(r_0 - r)\frac{e^2}{r} \right\| nl \right\rangle$.

4. Quark Effects in Magnetic Moments of Mirror Nuclei and Hypernuclei

4

QUARK EFFECTS IN MAGNETIC MOMENTS OF MIRROR NUCLEI AND HYPERNUCLEI

4.1 Schmidt Diagram and Quark Degrees of Freedom

In the traditional view of nuclear physics nucleons are the elementry constituents of nuclei. In this view the static properties of bound nucleons are the same as those determined for free nucleons. Consequently, the magnetic moment of the nucleus is simply the vectorial sum of magnetic moment of individual nucleons plus the orbital contribution of protons. For nuclei with a single nucleon outside the closed core or a single hole in an otherwise closed core, the magnetic moments are given by Schmidt values, for excess neutron

$$\mu_{Sch} = \pm \frac{2J}{(2l+1)}\mu_n' \qquad \Big| J = l \pm \tfrac{1}{2} \tag{4.1-1}$$

for excess proton

$$\mu_{Sch} = J\left[1 \pm \frac{(-1)}{(2l+1)}\right] \pm \frac{2J}{(2l+1)}\mu_p' \qquad \Big| J = l \pm \tfrac{1}{2} \tag{4.1-2}$$

In the hybrid quark model nucleons exhibit quark degrees of freedom and form a six-quark bag, when the separation between two nucleon is less than a certain critical radius r_0. When the nucleon resides in a six-quark bag, its effective magnetic moment is increased. This is natural in a bag model, where the magnetic moment of a (massless) quark is proportional to the radius of the bag (*Thomas*, 1983). In the same bag model when the number of quarks are doubled from three to six, the bag radius is increased by a factor of about 4/3. This factor results from the bag model virial theorem in which the mass of the bag is proportional to its volume. The mass of the six-quark bag is about twice that of a three-quark bag plus some 300MeV which accounts for the repulsive core. Therefore, in the quark picture the effective magnetic moment of the nucleon is larger by about a factor of 4/3 (*Karl et al*,1984). Thus, in the hybrid model framework the Schmidt expression for magnetic moment of nuclei can be expressed as,

for excess neutron

$$\mu_{Cal} = \pm \frac{2J}{(2l+1)}\left[\left(1+P_{NN}^{6q}(r_0)\right)\mu_n + P_{NN}^{6q}(r_0)\,\mu_n'\right] \quad \Big|\, J = l \pm \tfrac{1}{2} \qquad (4.1\text{-}3)$$

for excess proton

$$\mu_{Cal} = J\left[1 \pm \frac{(-1)}{(2l+1)}\right] \pm \frac{2J}{(2l+1)}\left[\left(1-P_{NN}^{6q}(r_0)\right)\mu_p + P_{NN}^{6q}(r_0)\,\mu_p'\right] \quad \Big|\, J = l \pm \tfrac{1}{2} \qquad (4.1\text{-}4)$$

$P_{NN}^{6q}(r_0)$ is the probability when the valence nucleon forms a six-quark bag with any of the core nucleons and $\mu_n'(\mu_p')$ are the magnetic moments of neutron (proton) inside a six-quark bag.

The isoscaler (μ^{IS}) and the isovector (μ^{IV}) components of magnetic moment for the mirror pair ${}^{A+1}_{z}X$ and ${}^{A+1}_{z+1}Y$ are defined as,

$$\mu^{IS} = \tfrac{1}{2}\left[\mu\left({}^{A+1}_{z+1}Y\right) + \mu\left({}^{A+1}_{z}X\right)\right],$$

$$\mu^{IV} = \tfrac{1}{2}\left[\mu\left({}^{A+1}_{z+1}Y\right) - \mu\left({}^{A+1}_{z}X\right)\right].$$

4.2 Quark Effects in Deformed Bags

Over the past few years the MIT bag model (*Chodos et al*, 1974 and *Degrand,* 1975) and its modifications (*Miller et al* , 1980,1981; *Vento et al* , 1980; *Barnhill et al,* 1980; *Huang and Stump,* 1976 and *Friedberg and Lee,* 1977, 1978) have been very successful in describing and explaining the properties of hadrons. In most of the bag model studies the bag is assumed to have a spherically symmetric shape. In order to account for certain discrepancies with the experimental data, some of the authors (*DeTar*, 1978, 1979; *Shanker et al*, 1982; *Viollier et al,* 1983, *Ma and Wambach*, 1983, *Hahn et al,* 1983 and *Brown*, 1982) have considered the formation of deformed bags with large D-state admixture in it. The formation of the deformed bags can be understood in the following way. As soon as the bag is populated by quarks in excited state or by gluons, these particles will exert a non-isotropic pressure on the bag surface and thus the bag gets deformed. Moreover, whenever hadrons come very close together, that is to the distances typically of the size of hadron, they may get polarized and deformed. The magnetic moment of a deformed nucleonic bag consisting of three quarks is different than the intrinsic magnetic moment of free nucleon and is dependent on the D-state admixture parameter. For single nucleons the magnetic moments can be expressed as (*Abbas* , 1987),

$$\mu(n) = -\frac{2}{3}\left[1 - P_D(N)\right]\mu_q \tag{4.2-1}$$

$$\mu(p) = \left[1 - P_D(N)\right]\mu_q \tag{4.2-2}$$

$\mu_q = e\hbar/2m_qc$, where m_q is the mass of the quark and $P_D(N)$ is the probability of D-state admixture in the nucleon bag. If the nucleon bags are deformed, the Schmidt value of the magnetic moments for a pair of mirror nuclei in the quark picture can be expressed as,

for neutron rich nucleus,

$$\mu\left({}^{A+1}_{z}X\right) = \pm \frac{2J}{(2l+1)}\left[1 - P_D\left({}^{A+1}_{z}X\right)\right]\mu_q \qquad \Big| J = l \pm ½ \tag{4.2-3}$$

for proton rich nucleus,

$$\mu\left({}^{A+1}_{z+1}Y\right) = J\left[1 \pm \frac{(-1)}{(2l+1)}\right] \pm \frac{2J}{(2l+1)}\left\{\left[1 - P_D\left({}^{A+1}_{z+1}Y\right)\right]\right\}\mu_q \qquad \Big| J = l \pm ½ \tag{4.2-4}$$

where $P_D\left({}^{A+1}_{z}X\right)$ and $P_D\left({}^{A+1}_{z+1}Y\right)$ refer to the deformation of the odd nucleon for a particular nucleus. For proton rich nucleus the contribution from the intrinsic part can be expressed as

$$\mu'\left({}^{A+1}_{z+1}Y\right) = \mu\left({}^{A+1}_{z+1}Y\right) - J\left[1 \pm \frac{(-1)}{(2l+1)}\right] \qquad \Big| J = l \pm ½ \tag{4.2-5}$$

The deformation parameter can P_D for a particular nucleus can be calculated from the known experimental values of the magnetic moments of nuclei using the following expressions

$$\frac{\mu\left({}^{A+1}_{z}X\right)}{\mu(n)} = \mp \frac{3J}{(2l+1)}\left[\frac{1 - P_D\left({}^{A+1}_{z}X\right)}{1 - P_D(N)}\right] \qquad \Big| J = l \pm ½ \tag{4.2-6}$$

$$\frac{\mu'\left({}^{A+1}_{z+1}Y\right)}{\mu(p)} = \pm \frac{2J}{(2l+1)}\left[\frac{1 - P_D\left({}^{A+1}_{z+1}Y\right)}{1 - P_D(N)}\right] \qquad \Big| J = l \pm ½ \tag{4.2-7}$$

The ratio of the deviations (δ) in the magnitude of the magnetic moments from the intrinsic magnetic moment of free nucleons in mirror nuclei is defined as,

$$\delta = \frac{\delta\mu\left({}^{A+1}_{z}X\right)}{\delta\mu\left({}^{A+1}_{z+1}Y\right)} = \frac{\mu\left({}^{A+1}_{z}X\right) - \mu(n)}{\mu'\left({}^{A+1}_{z+1}Y\right) - \mu(p)} \tag{4.2-8}$$

If magnetic moments of nuclei are expressed as in equations (4.2-3) and (4.2-5), the above equation can be rewritten as

$$\delta = -\frac{2}{3}\left[\frac{\pm 2J\left[1 - P_D\left({}^{A+1}_{z}X\right)\right] - (2l+1)\left[1 - P_D(N)\right]}{\pm 2J\left[1 - P_D\left({}^{A+1}_{z+1}Y\right)\right] - (2l+1)\left[1 - P_D(N)\right]}\right] \tag{4.2-9}$$

4.3 Magnetic Moments of Hypernuclei

Magnetic moments of light Λ-hypernuclei with the X–N–Λ configuration, where X,N and Λ denote the core $\left({}^{4}He, {}^{16}O \text{ and } {}^{40}Ca\right)$, a nucleon ($n$ and p) and a Λ-hyperon respectively are obtained by coupling a nuclear moment (μ_N) and Λ–hyperon moment (μ_Λ). The total hypernuclear spin J is obtained by coupling a nuclear spin j_N with a Λ-hyperon spin. If the hyperon is assumed to be in the $s_{1/2}$ state, then the hypernuclear ground state is simply described by $\left[j_N \otimes \Lambda s_{1/2}\right]_J$. Magnetic moments of the X–N–Λ system are given by

$$\mu_{Sch} = \frac{2j_N - 1}{2j_N} \cdot \frac{2j_N + 2}{2j_N + 1}\mu_N - \frac{2j_N - 1}{2j_N + 1}\mu_\Lambda \qquad \left|J = j_N - \tfrac{1}{2}\right. \tag{4.3-1}$$

and

$$\mu_{Sch} = \mu_N + \mu_\Lambda \qquad \left|J = j_N + \tfrac{1}{2}\right. \tag{4.3-2}$$

Where μ_N and μ_Λ denote a nuclear magnetic moment of j_N state and Λ-hyperon moment of the $s_{1/2}$ state respectively. In the hybrid quark model the

hyperon can from a six-quark bag with the core nucleons and the valence nucleons. If the average probability of the hyperon nucleon bag formation is $P_{\Lambda N}^{6q}(r_0)$ and the effective magnetic moments of hyperon inside the six-quark bag is μ'_Λ, then equations (4.3-1) and (4.3-2) get modified to,

$$\mu_{Cal} = \frac{2j_N - 1}{2j_N} \cdot \frac{2j_N + 2}{2j_N + 1}\mu_N - \frac{2j_N - 1}{2j_N + 1}\left[\left(1 - P_{\Lambda N}^{6q}(r_0)\right)\mu_\Lambda + P_{\Lambda N}^{6q}(r_0)\mu'_\Lambda\right] \quad \left| J = j_N - \tfrac{1}{2}\right. \tag{4.3-3}$$

and

$$\mu_{cal} = \mu_N + \left[\left(1 - P_{\Lambda N}^{6q}(r_0)\right)\mu_\Lambda + P_{\Lambda N}^{6q}(r_0)\mu'_\Lambda\right] \quad \left| J = j_N + \tfrac{1}{2}\right. \tag{4.3-4}$$

The magnetic moment of a hyperon inside a six-quark hyperon nucleon bag (μ'_Λ) can either be estimated empirically (MIT bag model) or microscopically (quark exchange current model). It has been shown (*Takeuchi et al,* 1988) that the effect of quark exchange between Λ-particle and a spin closed shell vanishes. For hypernuclei with X-N-Λ configuration the magnetic moment of a hyperon inside a six-quark bag formed with the valence nucleon can be estimated as described below. A hyperon can from a six-quark bag with a nucleon in both S=1 and S=0 configurations with statistical weights in the ratio of 3 : 1. The magnetic moment of hyperon inside a six-quak bag can be obtained by subtracting the magnetic moment of free nucleon from that of the six-quark hyperon-nucleon bag. Thus,

for hyperon-proton bag

$$\mu'_\Lambda = \frac{3}{4}\left[\mu(\Lambda\uparrow p\uparrow) - \mu(p\uparrow)\right] + \frac{1}{4}\left[\mu(\Lambda\uparrow p\downarrow) - \mu(p\downarrow)\right], \tag{4.3-5}$$

and for hyperon-neutron bag

$$\mu'_{\Lambda} = \frac{3}{4}\left[\mu(\Lambda\uparrow n\uparrow) - \mu(n\uparrow)\right] + \frac{1}{4}\left[\mu(\Lambda\uparrow n\downarrow) - \mu(n\downarrow)\right]. \tag{4.3-6}$$

For hypernuclei with closed core+Λ configuration, the magnetic moments in the extreme single-particle shell model, where Λ-hyperon is in the j state are given by the expressions

$$\mu_{Sch} = -\left(\frac{j}{j+1}\right)\mu_{\Lambda} \qquad |j = l - ½ \tag{4.3-7}$$

and

$$\mu_{Sch} = \mu_{\Lambda} \qquad |j = l + ½ \tag{4.3-8}$$

When the Λ-hyperon froms a six-quark bag with the core nucleons with a probability $P^{6q}_{\Lambda N}(r_0)$, equations (4.3-7) and (4.3-8) get modified to,

$$\mu_{cal} = -\left(\frac{j}{j+1}\right)\left[1 - P^{6q}_{\Lambda N}(r_0)\mu_{\Lambda} + P^{6q}_{\Lambda N}(r_0)\mu'_{\Lambda}\right] \qquad |j = l - ½ \tag{4.3-9}$$

and

$$\mu_{cal} = \left[\left(1 - P^{6q}_{\Lambda N}(r_0)\right)\mu_{\Lambda} + P^{6q}_{\Lambda N}(r_0)\mu'_{\Lambda}\right] \qquad |j = l + ½ \tag{4.3-10}$$

5. BINDING ENERGY : RESULTS AND DISCUSSIONS

5

BINDING ENERGY : RESULTS AND DISCUSSIONS

We have estimated the six-quark cluster effect in the Λ-binding energy difference of mirror hypernuclei pair (i) ${}^{6}_{\Lambda}He \sim {}^{6}_{\Lambda}Li$ and (ii) ${}^{14}_{\Lambda}C \sim {}^{14}_{\Lambda}N$. These are the two lightest pair of 1-p shell mirror hypernuclei with one nucleon outside the closed core. As shown in equation (3.1-8) the quark contribution to the binding energy difference depends on the six-quark probabilities $P^{6q}_{NN}(r_0)$ and $P^{6q}_{\Lambda N}(r_0)$. These in turn depend on the choice of wavefunction and the critical radius r_0 (equation (2.2-6)). For our calculations we have used harmonic oscillator wavefunctions to describe nucleons and hyperons in the hypernuclei. We have computed six-quark probabilities $P^{6q}_{NN}(r_0)$ and $P^{6q}_{\Lambda N}(r_0)$ by both the Slater and the Moshinsky methods as described in the formalism. Different authors have choosen different values for oscillator length parameters ν_N and ν_Λ for nucleon and hyperon respectively, based on various considerations. *Gal et al* (1971,1972) have assumed a uniform constant ν_N through out the p-shell and obtained the values of ν_N and ν_Λ from the best fit to the charge radii measured by electron scattering experiments (*Meyer–Berkhout et al*, 1959). *Mujib et al* (1979) have obtained the oscillater length parameter ν_N from the experimental values of the rms radii of the core nuclei and have fixed the hyperon oscillater length parameter ν_Λ by minimizing the Λ-particle binding energy in hypernuclei. *Wang and Wong* (1985) and *Wang et al* (1988) have determined the oscillater

length parameter from the experimental charge radius, the proton and the neutron charge radii after making a centre-of-mass correction,

In the calculation of probabilities by Slater method the values of ν_N and ν_Λ can be fixed independently of each other. However in Moshnisky method, to facilitate Moshnisky transformation to relative basis in the matrix elements in equation (2.2a-1) we have to choose $\nu_\Lambda = \left(m_\Lambda / m_N\right) \nu_N$. Thus fixing ν_N automatically fixes ν_Λ and vice versa. This prescription has been used earlier by *Bando et al* (1985) and *Mehrotra* (1991) in the study of hypernuclei. In Mujib I and Mujib II the oscillater length parameters ν_N and ν_Λ are obtained by fixing the value of one of the oscillater length parameters ν_N or ν_Λ and calculating the other from the above prescription. Different sets of values of oscillater length parameter used in our calculations are shown in **Table 5-1**. The values marked with * are the onces obtained by the above prescription and have been used by us in the calculation of probabilities by the Moshinsky method. As an excercise and for comparision purposes we have calculated the probability by Slater method also for the oscillater length parameter sets obtained by above prescription. For ${}^{6}_{\Lambda}He \sim {}^{6}_{\Lambda}Li$ pair the results of our calculations for six-quark probabilities are shown in **Table 5-2** for Moshinsky method and in **Table 5-3** for Slater method for the parameters of Gal I. In these tables $P^{6q}_{Q_1}(r_0)$ and $P^{6q}_{Q_2}(r_0)$ give the values of the exclusive probabilities for the formation of six-quark bags $\left(P^{6q}_{Q_1}(r_0)\right)$ and nine-quark bags $\left(P^{6q}_{Q_2}(r_0)\right)$ of the valence nucleon with one core nucleon and two core nucleons respectively and have been calculated using equations (2.2-15) and (2.2-16). The last column in the tables gives the value of the Coulomb energy lost (ΔV_c) by cutting the two proton integral off at distances $r < r_0$. In evaluating ΔV_c it is assumed that the Coulomb potential between the two protons does not change if one of the proton forms a six-quark bag with the neutron. Coulomb energy of two protons, when they are closer than r_0 should

Table 5-1

Values of oscillater length parameter for nucleon (ν_N) *and hyperon* (ν_Λ) *used by different authors. The values of* ν_Λ *marked with * are obtained using relation* $\nu_\Lambda = (m_\Lambda / m_N)\nu_N$.

Nuclei	Gal[a]		Mujib[b]		Wang[c]	
	ν_N (fm^{-2})	ν_Λ (fm^{-2})	ν_N (fm^{-2})	ν_Λ (fm^{-2})	ν_N (fm^{-2})	ν_Λ (fm^{-2})
${}^{6}_{\Lambda}He \sim {}^{6}_{\Lambda}Li$	**0.41** **0.41**	**0.49*** **0.33**	**0.272**	**0.312**	**0.323**	**0.384***
${}^{14}_{\Lambda}C \sim {}^{14}_{\Lambda}N$	**0.41** **0.41**	**0.49*** **0.330**	**0.367**	**0.207**	**0.385**	**0.457***

a) *Gal et al (1971, 1972).*

b) *Mujib et al (1979).*

c) *Wang and Wong (1985).*

Table 5-2

Average probability of the valence nucleon in ${}^{6}_{\Lambda}He \sim {}^{6}_{\Lambda}Li$ *to form a six-quark bag with the core nucleons* $(P^{6q}_{NN}(r_0))$ *and hyperon* $(P^{6q}_{\Lambda N}(r_0))$ *for different values of the cut off radius* r_0. $P^{6q}_{Q_1}(r_0)$ and $P^{6q}_{Q_2}(r_0)$ *are the six-quark and nine-quark bag formation probabilities with one and two core nucleons respectively.* ΔV_c *is the loss in Coulomb energy when the two protons overlap for* $r < r_0$. *The values shown have been calculated with the parameters of Gal I using Moshinsky method. (In Gal I ;* $\nu_N = 0.41\ fm^{-2}$, $\nu_\Lambda = 0.49\ fm^{-2}$*)*

r_0 (fm)	$P^{6q}_{NN}(r_0)$	$P^{6q}_{Q_1}(r_0)$	$P^{6q}_{Q_2}(r_0)$	$P^{6q}_{\Lambda N}(r_0)$	ΔV_C (MeV)
0.85	0.03105	0.03033	0.000119	0.00439	0.02091
0.87	0.03668	0.03568	0.000165	0.00520	0.02336
0.89	0.03673	0.03573	0.000166	0.00521	0.02337
0.91	0.04300	0.04163	0.000226	0.00612	0.02594
0.93	0.04306	0.04168	0.000227	0.00613	0.02595
0.95	0.04311	0.04173	0.000228	0.00614	0.02596
0.97	0.04994	0.04809	0.000304	0.00714	0.02862
0.99	0.04999	0.04814	0.000305	0.00715	0.02863
1.0	0.05004	0.04819	0.000306	0.00716	0.02864

Table 5-3

Six-quark bag probabilities ($P_{NN}^{6q}(r_0)$, $P_{Q_1}^{6q}(r_0)$, $P_{\Lambda N}^{6q}(r_0)$), nine-quark bag probabilities ($P_{Q_2}^{6q}(r_0)$) and loss in Coulomb energy (ΔV_c) for different values of the cut off raduis r_0 in ${}^{6}_{\Lambda}He \sim {}^{6}_{\Lambda}Li$. The values shown have been calculated with the parameters of Gal I using Slater method (In Gal I, $\nu_N = 0.41 fm^{-2}$, $\nu_N = 0.49 fm^{-2}$)

r_0 (fm)	$P_{NN}^{6q}(r_0)$	$P_{Q_1}^{6q}(r_0)$	$P_{Q_2}^{6q}(r_0)$	$P_{\Lambda N}^{6q}(r_0)$	ΔV_C (MeV)
0.85	0.05758	0.05513	0.000403	0.01915	0.03708
0.87	0.06854	0.06508	0.000567	0.02271	0.04194
0.89	0.06866	0.06518	0.000569	0.02274	0.04197
0.91	0.08108	0.07625	0.000789	0.02674	0.04721
0.93	0.08120	0.07636	0.000791	0.02678	0.04724
0.95	0.08133	0.07647	0.000794	0.02682	0.04726
0.97	0.09505	0.08844	0.001076	0.03121	0.05281
0.99	0.09518	0.08854	0.001079	0.03124	0.05284
1.0	0.09531	0.08866	0.001082	0.03128	0.05287

naturally be excluded, because it is already included in the six-quark bag mass. **Table 5-4** and **Table 5-5** show the contribution of direct and exchange terms to the six-quark probability. It is worth noting that the Pauli exchange terms in $P_{NN}^{6q}(r_0)$ is about 40% of the direct term and leads to a sizable reduction in the six-quark probability.

The six-quark probabilities $P_{NN}^{6q}(r_0)$ and $P_{\Lambda N}^{6q}(r_0)$ depend on the cut off radius r_0. Thus, it is necessary to use some constrain for the value of r_0. According to *Lomon* (1984) $r_0 < R_6$ (where R_6 is the radius of six-quark bag) to ensure that asymptotic freedom holds in the inner region. On the other hand, r_0 can not be too small since one expects asymptotic freedom to break down for $r > r_0$. Lomon used $r_0 \geq 0.8 R_6$. Combining these limits, $r_0 \approx 0.9 R_6$. Further, in the oscillater bag model $R_6 \approx 1.54 v^{-1/2}$. Thus r_0 ranges from 0.85 fm to $1.0 fm$. If $r_0 > 1.0 fm$, the conventional meson exchange picture of nuclear forces is difficult to understand. In most of the earlier studies (*Köch and Millor* , 1985; *Nag and Sural*, 1992 and *Heddle and Kisslinger* , 1986) a value of $r_0 = 1.0 fm$ has been preferred. The results for the six-quark probabilities for other sets of oscillater length parameters for Moshinsky and Slater method are shown in **Tables 5-6** and **Table 5-7** respectively for $r_0 = 1.0 fm$.

The difference in the mass of six-quark cluster for two neutrons and of two protons $(m_{nn} - m_{pp})$ is model dependent. Four sets of values computed by *Köch and Miller* (1985) in the nonrelativistic quark model (NRQM) and MIT bag model have been used in our calculations and are shown in **Table 5-8**. In NRQM the values are for three different sets of parameters of the strong interaction coupling constant α_s and the oscillator length parameter v describing the wave function.

Table 5-4

The direct and exchange term contribution to the average six-quark bag probabilities $P_{NN}^{6q}(r_0)$for the valence nucleon to form a six-quark bag with the core nucleon in $^{6}_{\Lambda}He \sim {}^{6}_{\Lambda}Li$ as a function of cut off radius r_0. The values shown have been calculated with Gal I parameters using Moshinsky method.

r_0 *(fm)*	Direct term	Exchange term
0.85	0.07904	0.03387
0.87	0.09336	0.04001
0.89	0.09349	0.04007
0.91	0.10947	0.04691
0.93	0.10960	0.04697
0.95	0.10973	0.04703
0.97	0.12712	0.05448
0.99	0.12725	0.05453
1.0	0.12738	0.05459

Table 5-5

The direct and exchange term contribution to the average six-quark bag probabilities $P_{NN}^{6q}(r_0)$ for the valence nucleon to form a six-quark bag with the core nucleon in ${}^{6}_{\Lambda}He \sim {}^{6}_{\Lambda}Li$ as a function of cut off radius r_0. The values shown have been calculated with Gal I parameters using Slater method.

r_0 (fm)	Direct term	Exchange term
0.85	0.01855	0.00832
0.87	0.02201	0.00975
0.89	0.02204	0.00976
0.91	0.02593	0.01132
0.93	0.02597	0.01133
0.95	0.02600	0.01134
0.97	0.03028	0.01302
0.99	0.03031	0.01303
1.0	0.03035	0.01304

Table 5-6

Average six-quark probability for the valence nucleon in ${}^{6}_{\Lambda}He \sim {}^{6}_{\Lambda}Li$ to form a six-quark bag with the core nucleons ($P^{6q}_{NN}(r_0)$) and hyperon ($P^{6q}_{\Lambda N}(r_0)$)for the cutoff radius $r_o = 1fm$. $P^{6q}_{Q_1}(r_0)$and $P^{6q}_{Q_2}(r_0)$ are the six-quark and nine-quark bag formation probabilities with one and two core nucleons respectively. $\Delta V_c$is the loss in Coulomb energy when the two protons overlap for $r < r_0$ The values shown have been calculated in Moshinsky method for different sets of oscillater length parameters (ν_N, ν_Λ).

Reference	$P^{6q}_{NN}(r_0)$	$P^{6q}_{Q_1}(r_0)$	$P^{6q}_{Q_2}(r_0)$	$P^{6q}_{\Lambda N}(r_0)$	ΔV_C (MeV)
Mujib I[a]	0.02793	0.02735	0.00010	0.00393	0.01588
Mujib II[b]	0.07969	0.07502	0.00076	0.01156	0.04596
Wang[c]	0.03572	0.03477	0.00016	0.00504	0.02036

a) $\nu_N = 0.272\ fm^{-2};\ \nu_\Lambda = 0.1477\ fm^{-2}$.

b) $\nu_N = 0.5746\ fm^{-2};\ \nu_\Lambda = 0.312\ fm^{-2}$.

c) $\nu_N = 0.323\ fm^{-2};\ \nu_\Lambda = 0.384\ fm^{-2}$.

Table 5-7

Six-quark bag probabilities ($P^{6q}_{NN}(r_0)$, $P^{6q}_{Q_1}(r_0)$, $P^{6q}_{\Lambda N}(r_0)$), nine-quark bag probabilities ($P^{6q}_{Q_2}(r_0)$) and loss in Coulomb energy (ΔV_c) of ${}^{6}_{\Lambda}He \sim {}^{6}_{\Lambda}Li$ obtained by Slater method for the cut off radius $r_0 = 1fm$. The values shown have been calculated for different sets of oscillater length parameters (ν_N, ν_Λ) used by different authors.

Reference	$P^{6q}_{NN}(r_0)$	$P^{6q}_{Q_1}(r_0)$	$P^{6q}_{Q_2}(r_0)$	$P^{6q}_{\Lambda N}(r_0)$	ΔV_C *(MeV)*
Gal II[a]	0.09531	0.08866	0.00108	0.02845	0.05287
Mujib[b]	0.05153	0.04956	0.00032	0.01709	0.02796
Wang[c]	0.06668	0.06340	0.00054	0.02209	0.03649

a) $\nu_N = 0.41\ fm^{-2};\ \nu_\Lambda = 0.33\ fm^{-2}$.

b) $\nu_N = 0.272\ fm^{-2};\ \nu_\Lambda = 0.312\ fm^{-2}$.

c) $\nu_N = 0.323\ fm^{-2};\ \nu_\Lambda = 0.384\ fm^{-2}$.

Table 5-8

Mass difference of six-quark bag for two neutrons and two protons in different models.

S.No	Model	$m_{nn} - m_{pp}$ * (MeV)
1.	NRQM I	-1.86
2.	NRQM II	-1.45
3.	NRQM III	- 0.58
4.	MIT Bag	0.432

* *Köch and Miller (1985)*

In Model I the parameters are obtained by fitting the Δ-N mass splitting and energy levels of the nonstrange supermultiplets. In model II the parameter are obtained by including contribution of pion cloud effects to the Δ-N mass splitting. The parameters of Model III are fixed from the experimental values of proton rms charge radius. In the nonrelativistic quark model the two proton bag is heavier than two neutron bag. The mass difference of six-quark cluster of neutron-hyperon and proton-hyperon $(m_{n\Lambda} - m_{p\Lambda})$ calculated using equation (3.2b-9). With $C_d - C_u = 4\text{MeV}$ and $R_6 = 1.2fm$ the differences is 1.3998 MeV. The MIT bag model value is much smaller than the free value of 2.6 MeV.

Using the calculated values of $P^{6q}_{NN}(r_0)$, $P^{6q}_{\Lambda N}(r_0)$, $m_{n\Lambda} - m_{p\Lambda}$, ΔV_c and NRQM and MIT bag model values of $m_{nn} - m_{pp}$, we have estimated the six-quark cluster contribution to the binding energy difference between ${}^{6}_{\Lambda}He \sim {}^{6}_{\Lambda}Li$ for different cases. The results are summarized in **Table 5-9** and **Table 5-10**, for $r_0 = 1.0fm$. The six-quark probability $P^{6q}_{NN}(r_0)$ is strongly dependent on the choice of the parameters and ranges between 3% to 10% for $r_0 = 1.0fm$. $P^{6q}_{\Lambda N}(r_0)$ is much smaller and lies between 0.4% to 1%. In all the calculations the six-quark cluster formation probabilities obtained in Moshinsky method are smaller than the corresponding values obtained in Slater method. This is in accordance with the calculation of Kang and Oshagan (*Wang et al* , 1988). These authors have shown that the value of six-quark probability calculated by using Moshinsky method are smaller than those calculated by Slater method. The results of our calculation show that the quark contribution to the binding energy is sizable and varies between 14keV to 157keV. The split in various terms contributing to the quark correction to the binding energy difference is shown in **Table 5-11** for the parameters of Gal I. If the exclusive probabilities $P^{6q}_{Q_1}(r_0)$ and $P^{6q}_{Q_2}(r_0)$ defined in equations (2.2-15) and (2.2-16) are used and the nine-quark bag

Table 5-9

*Six-quark cluster contribution to the Λ-binding energy difference (ΔB_{6q}) between mirror hypernuclei pair ${}^{6}_{\Lambda}He$ ~ ${}^{6}_{\Lambda}Li$ for the cut off radius $r_0 = 1fm$. The results are shown for different sets of oscillater length parameters from **Table 5-6** and $m_{nn} - m_{pp}$ mass difference from **Table 5-8** using Moshinsky method.*

Model	ΔB_{6q} (keV)			
	Gal I	Mujib I	Mujbi II	Wang
NRQM I	82	46	130	58
NRQM II	72	40	114	51
NRQM III	50	28	80	36
MIT Bag	25	14	39	18

Table 5-10

Six-quark cluster contribution to the Λ–binding energy difference (ΔB_{6q}) between mirror hypernuclei pair ${}^{6}_{\Lambda}He$ ~ ${}^{6}_{\Lambda}Li$ for the cut off radius $r_0 = 1fm$. The results are shown for different sets of oscillater length parameters from ***Table 5-7*** *and $m_{nn} - m_{pp}$ mass difference from* ***Table 5-8*** *using Slater method.*

Model	ΔB_{6q} (keV)			
	Gal I	Gal II	Mujib	Wang
NRQM I	157	157	85	110
NRQM II	137	137	75	96
NRQM III	96	96	52	67
MIT Bag	47	48	26	34

Table 5-11

Split of various terms in the quark contribution to the Λ-binding energy difference (ΔB_{6q}) of ${}^{6}_{\Lambda}He$ ~ ${}^{6}_{\Lambda}Li$ for r_0=1fm. The values shown have been calculated using Gal I parameters and NRQM I model for mass difference of six-quark bag of two neutrons and two protons.

Terms Contributing to ΔB_{6q}	NRQM I	
	MOSHINSKY METHOD (MeV)	SLATER METHOD (MeV)
$\frac{1}{2} P_{NN}^{6q}(r_0)\,(2m_n - 2m_p)$	0.0651	0.1239
$\frac{1}{2} P_{NN}^{6q}(r_0)\,(m_{pp} - m_{nn})$	+ 0.0465	+ 0.0886
$P_{\Lambda N}^{6q}(r_0)\,(m_n - m_p)$	+ 0.0093	+ 0.0407
$P_{\Lambda N}^{6q}(r_0)\,(m_{p\Lambda} - m_{n\Lambda})$	–0.0100	–0.0438
ΔV_C	–0.0286	–0.0529

masses are computed from equation (3.2b-9), the contribution to the binding energy difference arising from six-quark and nine-quark bags (equation 3.1-12) are 0.0522MeV and 0.0004MeV respectively in the Moshinsky method. The corresponding values are 0.0961 MeV and 0.0015MeV in the Slater method. Thus the dominant contribution to the binding energy difference comes from the exclusive six-quark probability.

We have made similar calculation for ${}^{14}_{\Lambda}C \sim {}^{14}_{\Lambda}N$ hypernuclei pair. The results for the six-quark bag formation probabilities and various other terms are summarized in **Table 5-12–5-15** for different cases for both Moshinsky and Slater methods. The results for the quark contributions to binding energy are given in **Table 5-16** and **Table 5-17**. Charge symmetry of Λ-N interaction leads to the expectation that Λ-particle should have equal binding energy in mirror pair of hypernuclei. The experimental data for the Λ-binding energy in ${}^{6}_{\Lambda}He \sim {}^{6}_{\Lambda}Li$ and ${}^{14}_{\Lambda}C \sim {}^{14}_{\Lambda}N$ are summarized in **Table 5-18** and **Table 5-19** respectively.

In the case of ${}^{6}_{\Lambda}He \sim {}^{6}_{\Lambda}Li$ the difference in the Λ-binding energy is nearly 250keV (*Hiyama et al*, 1996). The experimental value of Coulomb displacement energy for the mirror pair ${}^{6}_{\Lambda}He \sim {}^{6}_{\Lambda}Li$ is not known. If we add our calculated value of 468keV for $r_0 = 1.0fm$ the discrepancy increases to a substantial value of 718keV indicating a large violation of charge symmetry breaking effect. For the ${}^{14}_{\Lambda}C \sim {}^{14}_{\Lambda}N$ pair also, the experimental value of Coulomb displacement energy is not known. The experimental difference in the Λ-binding energy in ${}^{14}_{\Lambda}C \sim {}^{14}_{\Lambda}N$ ranges between 0 to 1.5 MeV. In our calculation the Coulomb energy for $r_0 = 1.0fm$ and for the parameters of Gal I is 1.859MeV. This can be compared with the value of 3.038MeV obtained by Wang for A=13 mirror nuclei in RGM calculation (*Wang et al,* 1988). If the Coulomb correction is added to the experimental values, Λ-particle becomes more bound in ${}^{14}_{\Lambda}N$ than in ${}^{14}_{\Lambda}C$ by nearly 1.86MeV, again showing a large charge symmetry breaking effect.

Table 5-12

Average probabilities for the valence nucleon in ${}^{14}_{\Lambda}C \sim {}^{14}_{\Lambda}N$ to form six-quark bag with the core nucleons ($P^{6q}_{NN}(r_0)$) and hyperon ($P^{6q}_{\Lambda N}(r_0)$)for different values of the cut off radius r_0.$P^{6q}_{Q_1}(r_0)$ and $P^{6q}_{Q_2}(r_0)$ are the six-quark and nine-quark bag formation probabilities with one and two core nucleons respectively. ΔV_c is the loss in Coulomb energy when the two protons overlap for $r < r_0$. The values shown have been calculated with the parameters of Gal I using Moshinsky method. (In Gal I $\nu_N = 0.41 fm^{-2}$; $\nu_\Lambda = 0.49 fm^{-2}$)

r_0 (fm)	$P^{6q}_{NN}(r_0)$	$P^{6q}_{Q_1}(r_0)$	$P^{6q}_{Q_2}(r_0)$	$P^{6q}_{\Lambda N}(r_0)$	ΔV_C (MeV)
0.85	0.06744	0.06339	0.000179	0.00879	0.04864
0.87	0.07989	0.07423	0.000248	0.01041	0.05517
0.89	0.08000	0.07433	0.000249	0.01043	0.05520
0.91	0.09395	0.08617	0.000340	0.01225	0.06222
0.93	0.09407	0.08627	0.000341	0.01226	0.06226
0.95	0.09420	0.08638	0.000342	0.01228	0.06230
0.97	0.10946	0.09896	0.000455	0.01428	0.06974
0.99	0.10958	0.09906	0.000456	0.01429	0.06978
1.0	0.10970	0.09916	0.000457	0.01431	0.06982

Table 5-13

Six-quark probabilities ($P^{6q}_{NN}(r_0)$, $P^{6q}_{Q_1}(r_0)$, $P^{6q}_{\Lambda N}(r_0)$), nine-quark probabilities ($P^{6q}_{Q_2}(r_0)$) and loss in Coulomb energy (ΔV_c) of ${}^{14}_{\Lambda}C$ ~ ${}^{14}_{\Lambda}N$ for different values of the cut off radius r_0. The values shown have been calculated with the parameters of Gal I using Slater method.

r_0 (fm)	$P^{6q}_{NN}(r_0)$	$P^{6q}_{Q_1}(r_0)$	$P^{6q}_{Q_2}(r_0)$	$P^{6q}_{\Lambda N}(r_0)$	ΔV_C (MeV)
0.85	0.07678	0.07154	0.000230	0.01915	0.06373
0.87	0.09385	0.08608	0.000339	0.02271	0.07517
0.89	0.09410	0.08630	0.000341	0.02274	0.07530
0.91	0.11430	0.10288	0.000495	0.02674	0.08834
0.93	0.11458	0.10311	0.000497	0.02678	0.08848
0.95	0.11488	0.10334	0.000499	0.02682	0.08862
0.97	0.13799	0.12150	0.000707	0.03121	0.10311
0.99	0.13830	0.12174	0.000710	0.03124	0.10325
1.0	0.13862	0.12199	0.000713	0.03128	0.10340

Table 5-14

Average probabilities for the valence nucleon in ${}^{14}_{\Lambda}C \sim {}^{14}_{\Lambda}N$ *to form six-quark bag with the core nucleons* $(P^{6q}_{NN}(r_0))$ *and hyperon* $(P^{6q}_{\Lambda N}(r_0))$ *for the cut off radius* $r_0 = 1fm$. *The values shown have been obtained using different sets of oscillater length parameter* (ν_N, ν_Λ) *in Moshinsky method.*

Reference	$P^{6q}_{NN}(r_0)$	$P^{6q}_{Q_1}(r_0)$	$P^{6q}_{Q_2}(r_0)$	$P^{6q}_{\Lambda N}(r_0)$	ΔV_C *(MeV)*
Mujib I [a]	0.09355	0.08583	0.000337	0.01217	0.05865
Mujib II [b]	0.09873	0.09016	0.000374	0.01285	0.06221
Wang [c]	0.10023	0.09139	0.000385	0.01303	0.06324

a) $\nu_N = 0.367\ fm^{-2};\ \nu_\Lambda = 0.1993\ fm^{-2}$.

b) $\nu_N = 0.381\ fm^{-2};\ \nu_\Lambda = 0.207\ fm^{-2}$.

c) $\nu_N = 0.385\ fm^{-2};\ \nu_\Lambda = 0.209\ fm^{-2}$.

Table 5-15

Six-quark probabilities $(P_{NN}^{6q}(r_0), P_{\Lambda N}^{6q}(r_0)\ , P_{Q_1}^{6q}(r_0))$*, nine-quark probabilities* $(P_{Q_2}^{6q}(r_0))$ *and loss in Conlomb energy* (ΔV_c) *of* ${}^{14}_{\Lambda}C \sim {}^{14}_{\Lambda}N$ *obtained by Slater Method for the cut off radius* $r_0 = 1 fm$*. The values shown have been calculated for different sets of oscillater length parameters* (v_N, v_Λ) *used by different authors.*

Reference	$P_{NN}^{6q}(r_0)$	$P_{Q_1}^{6q}(r_0)$	$P_{Q_2}^{6q}(r_0)$	$P_{\Lambda N}^{6q}(r_0)$	ΔV_C (MeV)
Gal II [a]	0.13862	0.12199	0.000713	0.02845	0.10340
Mujib [b]	0.11388	0.10254	0.000491	0.02040	0.08326
Wang [c]	0.12396	0.11058	0.000577	0.02853	0.09141

a) $v_N = 0.41\ fm^{-2};\ v_\Lambda = 0.33\ fm^{-2}.$

b) $v_N = 0.367\ fm^{-2};\ v_\Lambda = 0.207\ fm^{-2}.$

c) $v_N = 0.385\ fm^{-2};\ v_\Lambda = 0.4574\ fm^{-2}.$

Table 5-16

Six-quark cluster contribution to the Λ-binding energy difference (ΔB_{6q}) between mirror hypernuclei pair ${}^{14}_{\Lambda}C \sim {}^{14}_{\Lambda}N$ for the cut off radius $r_0 = 1fm$. The results are shown for different sets of oscillater length parameters obtained by different authors, and mass difference of six-quark bags for two neutrons and two protons in different models using Moshinsky method.

Model	ΔB_{6q} (keV)			
	Gal I	Mujib I	Mujib II	Wang
NRQM I	173	149	157	159
NRQM II	151	130	136	138
NRQM III	103	89	93	95
MIT Bag	48	42	43	44

Table 5-17

Six-quark cluster contribution to the Λ-binding energy difference (ΔB_{6q}) between mirror hypernuclei pair ${}^{14}_{\Lambda}C \sim {}^{14}_{\Lambda}N$ for the cut off radius $r_0 = 1fm$. The results are shown for different sets of oscillater length parameters obtained by different authors and mass difference of six-quark bags for two neutrons and two protons in different models using Slater method.

Model	ΔB_{6q} (keV)			
	Gal I	Gal II	Mujib	Wang
NRQM I	202	203	169	182
NRQM II	174	175	145	157
NRQM III	113	114	96	103
MIT Bag	43	44	38	40

Table 5-18

Experimental data of Λ-binding energy in ${}^{6}_{\Lambda}He$ and ${}^{6}_{\Lambda}Li$.

$B_{\Lambda}({}^{6}_{\Lambda}He)$ (MeV)	$B_{\Lambda}({}^{6}_{\Lambda}Li)$ (MeV)	Reference
4.19 ± 0.17	—	Daris and Sacton (1967)
—	5.89 ± 0.37	Harmsen (1967)
4.09 ± 0.27	—	Gajewski et al (1967)
—	3.92 ± 0.37	Harmsen (1967)
4.28 ± 0.15	5.55	Bhaduri and Nogami (1972)
4.25	4.50	Motoba et al (1985)
4.18 ± 0.10	4.50	Bando et al (1990)
4.18	4.50	Hiyama et al(1996)

Table 5-19

Experimental data of Λ-binding energy in ${}^{14}_{\Lambda}C$ and ${}^{14}_{\Lambda}N$.

$B_{\Lambda}({}^{14}_{\Lambda}C)$ (MeV)	$B_{\Lambda}({}^{14}_{\Lambda}N)$ (MeV)	Reference
13.2 ± 0.7	11.7±05	Iwao (1971)
12.17 ± 0.33	—	Mujib et al (1979)
13.2 ± 0.7	11.7 ± 0.5	Khan (1981)
12.17 ± 0.33	11.7 ± 0.5	Shoeb and Rahman (1984)
12.17 ± 0.33	—	Ahmad et al (1985)
$12.17 + 0.33$	11.77 ± 0.5	Rahman et al (1986)
12.17 ± 0.33	12.17 ± — —	Bando et al (1990)

The results of our calculations show that six-quark cluster formation effect increases the binding of Λ-hyperon in the neutron rich partner compared to that of its proton rich partner. The Λ-particle is more bound in ${}^{6}_{\Lambda}He$ and ${}^{14}_{\Lambda}C$ compared to that ${}^{6}_{\Lambda}Li$ and ${}^{14}_{\Lambda}N$ respectively. The reason being the color magnetic hyperfine interaction between quarks make $m_{nn} - m_{pp}$ less than the corresponding term $2m_n - 2m_p$ for free nucleons. This is in accordance with the observation made in *Greben and Thomas* (1984) and *Köch and Miller* (1985) but does not give the right sign for the experimental discrepancy.

It is worth mentioning that $P^{6q}_{NN}(r_0)$ and $P^{6q}_{\Lambda N}(r_0)$ are dependent on the choice of oscillater length parameters and increase with an increase of the values of v_N and v_Λ. **Figure I** and **Figure II** show the general trend of variation of $P^{6q}_{NN}(r_0)$ with v_N for A = 6 and A = 14 mirror hypernuclei pair respectively. The variation is shown for the parameters of Gal I for both Moshinsky and Slater methods. In the case of ${}^{14}_{\Lambda}C \sim {}^{14}_{\Lambda}N$ pair overlap probability of the valence nucleon with the $0s_{1/2}$ core nucleon and $0p_{3/2}$ core nucleons are presented separately in **Figure III.** As expected for $0p_{1/2}$ valence nucleon overlap probability with the s-core nucleon is larger compared to that with the p-core nucleons. In **Table 5-20** we compare the values of six-quark probabilities obtained by different authors with the results obtained in the present work. Only the values obtained by *Köch and Miller* (1985) for A=3 system are obtained from the experimentally measured ${}^{3}H \sim {}^{3}He$ electron scattering form factor in a model independent way. In all other calculations $P^{6q}_{NN}(r_0)$ has been evaluated phenomenologically and depends on the choice of the wave functions and r_0.

The results of our calculation show that the six-quark bag formation effect contributes significantly to the binding energy difference of the mirror pair of nuclei. The overlap probability of the valence nucleon with the hyperon also make a small contribution to the binding energy difference

Figure I

Variation of six-quark probability $P^{6q}_{NN}(r_0)$ with oscillater length parameter ν_N for $A = 6$ hypernuclei with cut off radius $r_0 = 1 fm$.

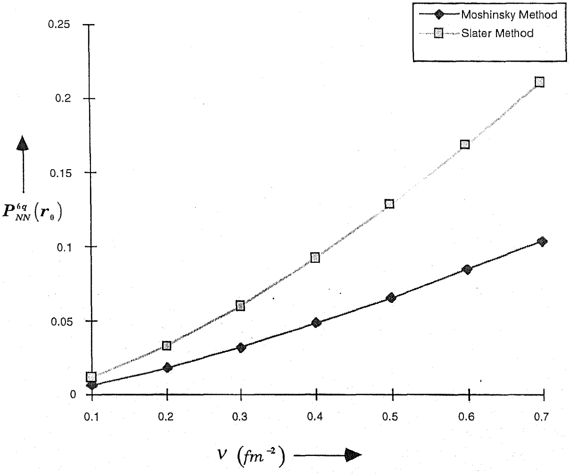


Figure II

Variation of six-quark probability $P_{NN}^{6q}(r_0)$ *with oscillater length parameter* ν_N *for A = 14 hypernuclei with cut off radius* $r_0 = 1fm$.

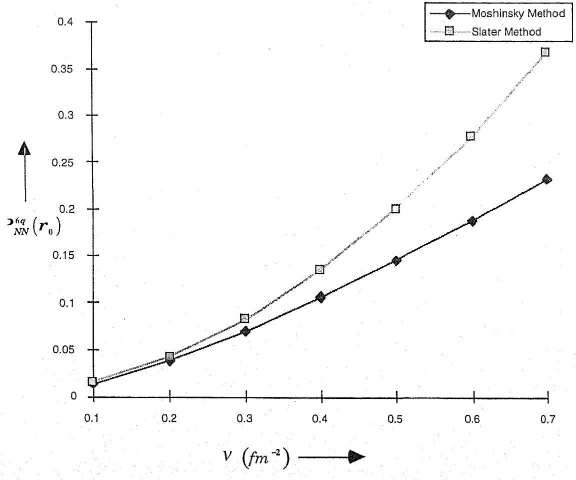


Figure III

Variation of six-quark probability $P^{6q}_{NN}(r_0)$ of valence nucleon with $0s_{1/2}$ and $0p_{3/2}$ core nucleons, with oscillater length parameter ν_N for A = 14 hypernuclei with cut off radius $r_0 = 1 fm$.

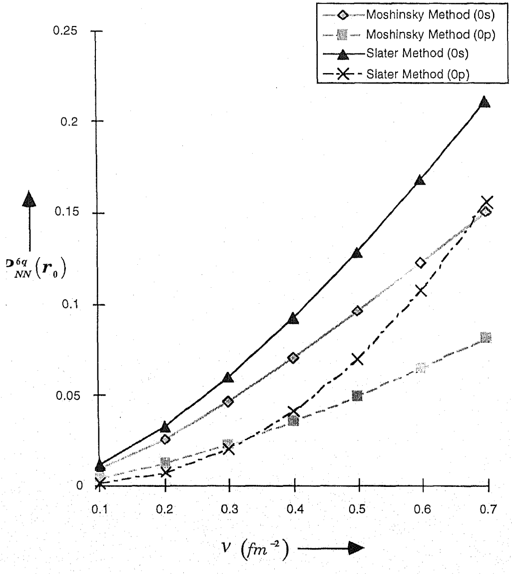


Table 5-20

Six-quark bag formation probabilities calculated by different authors for various mirror nuclei pair. The values of cut off radius r_0 are shown in column 2.

Mirror Pair	r_0 *(fm)*	$P_{NN}^{6q}(r_0)$	Reference
$^3H \sim {}^3He$	1.0	0.064	Köch and Miller (1985)
	0.95	0.148	Greben and Thomas (1984)
	1.0	0.15	Heddle and Kisslinger (1986)
	1.0	0.15	Pirner and Vary (1981)
	1.0	0.11	Vary (1983)
	1.0	0.03	Beyer and Weber (1986)
	1.0	0.13	Bhaduri et al (1986)
	1.0	0.09	Kalashnikova et al (1986, 1987)
$^{13}C \sim {}^{13}N$	0.95	0.151	Gerben and Thomas (1984)
${}^4_\Lambda H \sim {}^4_\Lambda He$	1.0	0.034	Nag and Sural (1992)
${}^6_\Lambda He \sim {}^6_\Lambda Li$	1.0	0.03 – 0.01	Present work
${}^{14}_\Lambda C \sim {}^{14}_\Lambda N$	1.0	0.09 – 0.14	Present work

and should be included in any reliable calculation. However quark effect makes Λ-hyperon more bound in the neutron rich partner than its proton rich partner and do not give the correct sign to the NS anomaly. Similar results have been obtained by *Köch and Miller* (1985) and *Greben and Thomas* (1984). It is worth mentioning that *Nag and Sural* (1992) have estimated the quark contribution to the binding energy difference of ${}^4_\Lambda H \sim {}^4_\Lambda He$ and have obtained the right sign to the NS anomaly of the above pair. We however, feel that the right sign is due to the particular values of P_{6q} and $\overline{P}_{6q}$ used in their calculation. Only P_{6q} is related to the experimental data but $\overline{P}_{6q}$ is strongly dependent on the wave functions and other parameters.

In conclusion we can say that the six-quark cluster formation effects contribute significantly to the binding energy of mirror pair of nuclei. Reliable estimate of these contributions depends on nuclear wavefunctions needed as input to determine the matrix element in P_{6q}. Only for A=3 nuclei P_{6q} has been obtained directly from experimental data. We need a detailed description of the conventional wave function and a better study of various other contributions to the binding energy difference to understand the large charge symmetry violation in the experimental binding energy of mirror nuclei.

6. MAGNETIC MOMENT : RESULTS AND DISCUSSIONS

6

MAGNETIC MOMENT : RESULTS AND DISCUSSIONS

6.1 Schmidt Values

In the present work we have studied the effect of quark degrees of freedom to account for the observed discrepancy from the Schmidt values for nuclei with one excess nucleon outside a closed core. We have made specific calculations for A=13, 17, 29, 33, 41 mirror pair of nuclei. The magnetic moment of these nuclei fall outside the Schmidt lines. In view of our understanding of nucleon as composite objects consisting of quarks it is natural to expect that nucleons change their properties inside the nuclei (*Ericson*, 1971 and *Noble* , 1981). This conclusion is suggested in **Table 6.1-1** and **Table 6.1-2** where we have displayed the effective magnetic moments of protons and neutrons as deduced from the experimental values of magnetic moment (*Aizenberg-Selove,* 1986; *Endt and Leun,* 1978; *Lederer and Shirley,* 1978 and *Raghavan*, 1989) of the above nuclei. These effective magnetic moment differ from their corresponding values for free nucleons.

In the hybrid quark model approach the magnetic moments of nuclei with one nucleon outside a closed core depends on the probability of the formation of the six-quark bag of the valence nucleon with the core nucleon and the effective magnetic moment of nucleon inside the bag (equations

Table 6.1-1

Intrinsic proton magnetic moment in light nuclei.

Nucleus	J^{π}	Observed * moment (n.m.)	Intrinsic moment (μ'_p) (n.m.)	$\Delta\mu$ (n.m.)
p	$1/2^+$	2.79	2.79	-
^{13}N	$1/2^-$	-0.3221	2.97	0.18
^{17}F	$5/2^+$	4.722	2.72	-0.07
^{29}P	$1/2^+$	1.2349	1.23	-1.56
^{33}Cl	$3/2^+$	0.7523	1.75	-1.04
^{41}Sc	$7/2^-$	5.430	2.43	-0.36

* *Raghavan (1989).*

Table 6.1-2

Intrinsic neutron magnetic moment in light nuclei.

Nucleus	J^π	Observed* moment (n.m.)	Intrinsic moment (μ_n') (n.m.)	$\Delta\mu$ (n.m.)
n	$\frac{1}{2}^+$	−1.91	−1.91	-
^{13}C	$\frac{1}{2}^-$	0.7024	−2.11	−0.20
^{17}O	$\frac{5}{2}^+$	−1.8937	−1.89	0.02
^{29}Si	$\frac{1}{2}^+$	−0.553	−0.55	1.36
^{33}S	$\frac{3}{2}^+$	0.6438	−1.07	0.83
^{41}Ca	$\frac{7}{2}^-$	−1.595	−1.60	0.31

* *Raghavan (1989).*

(4.1-3) and (4.1-4)). In our calculations we have used the values of six-quark probabilities as calculated by *Köch and Miller* (1985) and have taken the effective magnetic moment of nucleon inside the bag to be 4/3 times its value for the free nucleon. The results of numerical calculations for different nuclei are shown in **Table 6.1-3**. **Figure IV** and **Figure V** show the plot of Schmidt lines, experimental values and the values calculated in the present work for neutron rich and proton rich nuclei respectively. It is observed that the quark bag formation effect contributes significantly and tends to increase the magnetic moment of nuclei from their single particle values.

Isoscaler and isovector components of the experimental, Schmidt and calculated magnetic moments for the mirror nuclei are shown in **Table 6.1-4** and **Table 6.1-5** respectively. Large deviations are found in values of the isoscaler and isovector components of A=29, 33 pair. It has long been recognized that there are several other mechanisms which play important role in explaining the deviation of magnetic moments from the Schmidt values. These are the corrections arising due to mixing of configurations, core polarization, meson exchange current, quark exchange current and short-range correlation effects which have been studied in the literature. There have been accumulating evidences for the mixed configuration of nuclear states such as the beta-transitions with anomalous *ft* values, empirically found smooth variations of the first excited states of even-even nuclei, etc. *Arima and Horie* (1954) and independently *Blin-stoyle* (1953) have estimated the correction to the Schmidt values of magnetic moments, by considering the modification of the nuclear wavefunction via mixing of configurations. These corrections are referred to as $\delta\mu_{CM}$. Particle hole excitation between major shells also contribute to the correction to the magnetic moments (*Kitagawa,* 1999). Such corrections

Table 6.1-3

Magnetic moments of light mirror nuclei A=13, 17, 29, 33, 41. μ_{Sch} are the Schmidt values, μ_{Cal} are the values calculated including the six-quark cluster contribution with $\mu_N' = \frac{4}{3}\mu_N$, and μ_{Exp} refers to the experimental values. $P_{NN}^{6q}(r_0)$ are the six-quark bag formation probability of the valence nucleon with the core nucleons for cut off radius $r_0 = 0.95\, fm$.

A	J^π	$P_{NN}^{6q}(r_0)^a$	Nucleus	$\mu_{Exp}{}^b$ (n.m.)	μ_{Sch} (n.m.)	μ_{Cal} (n.m.)
13	$\frac{1}{2}^-$	0.151	${}^{13}_{6}C_7$	0.7024	0.6377	0.6687
			${}^{13}_{7}N_6$	−0.3221	−0.2633	−0.3101
17	$\frac{5}{2}^+$	0.117	${}^{17}_{8}O_9$	−1.8937	−1.913	−1.9845
			${}^{17}_{9}F_8$	4.722	4.79	4.8988
29	$\frac{1}{2}^+$	0.168	${}^{29}_{14}Si_{15}$	−0.553	−1.913	−2.0169
			${}^{29}_{15}P_{14}$	1.2349	2.79	2.9462
33	$\frac{3}{2}^+$	0.167	${}^{33}_{16}S_{17}$	0.6438	1.1478	1.2098
			${}^{33}_{17}Cl_{16}$	0.7523	0.126	0.0328
41	$\frac{7}{2}^-$	0.137	${}^{41}_{20}Ca_{21}$	−1.595	−1.913	−1.997
			${}^{41}_{21}Sc_{20}$	5.430	5.79	5.9174

a) *Köch and Miller (1985).*

b) *Raghavan (1989).*

Table 6.1-4

Isoscaler component of the experimental (μ^{IS}_{Exp}),*Schmidt* (μ^{IS}_{Sch})*and calculated* (μ^{IS}_{Cal}) *values of the magnetic moments for different nuclei. The difference between* (μ^{IS}_{Exp}) *and* (μ^{IS}_{Sch}) *is given by* $\Delta\ \mu^{IS}_{Sch}$ *and that between* μ^{IS}_{Exp} *and* μ^{IS}_{Cal} *is given by* $\Delta\ \mu^{IS}_{Cal}$.

A	μ^{IS}_{Exp} (n.m.)	μ^{IS}_{Sch} (n.m.)	μ^{IS}_{Cal} (n.m.)	$\Delta\ \mu^{IS}_{Sch}$ (n.m.)	$\Delta\ \mu^{IS}_{Cal}$ (n.m.)
13	0.1902	0.1872	0.1793	+0.0030	+0.0109
17	1.4142	1.4385	1.4572	–0.0243	–0.0430
29	0.3410	0.4385	0.4647	–0.0975	–0.1237
33	0.6981	0.6369	0.6213	+0.0612	+0.0768
41	1.9175	1.9385	1.9602	–0.0210	–0.0427

Table 6.1-5

Isovector component of the experimental (μ^{IV}_{Exp}), *Schmidt* (μ^{IV}_{Sch})*and calculated* (μ^{IV}_{Cal}) *values of the magnetic moments for different nuclei. The difference between* μ^{IV}_{Exp} *and* μ^{IV}_{Sch} *is given by* $\Delta\,\mu^{IV}_{Sch}$ *and that between* μ^{IV}_{Exp} *and* μ^{IV}_{Cal} *is given by* $\Delta\,\mu^{IV}_{Cal}$.

A	μ^{IV}_{Exp} (n.m.)	μ^{IV}_{Sch} (n.m.)	μ^{IV}_{Cal} (n.m.)	$\Delta\,\mu^{IV}_{Sch}$ (n.m.)	$\Delta\,\mu^{IV}_{Cal}$ (n.m.)
13	–0.5123	–0.4505	–0.4894	–0.0618	–0.0229
17	+3.3077	+ 3.3515	+ 3.4417	–0.0438	–0.01340
29	+0.8940	+2.3515	+2.4816	–1.4575	–1.5876
33	+0.0543	–0.0109	–0.5885	+0.0652	+0.6428
41	+3.5125	+3.8515	+3.9572	–0.3390	–0.4447

Figure IV

Variation of magnetic moments (μ) of neutron rich partner of the mirror nuclei pair with mass number (A). The plots are for experimental, Schmidt and calculated values in the present work.

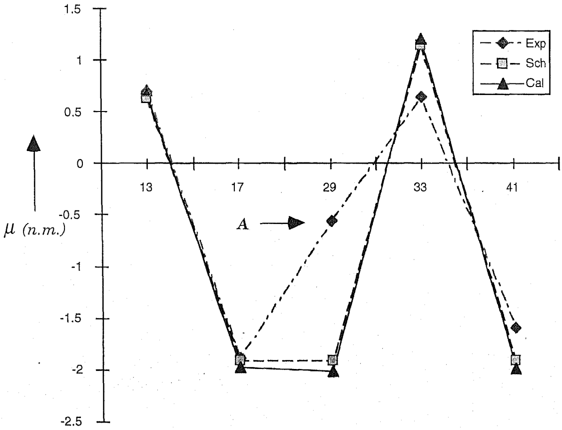
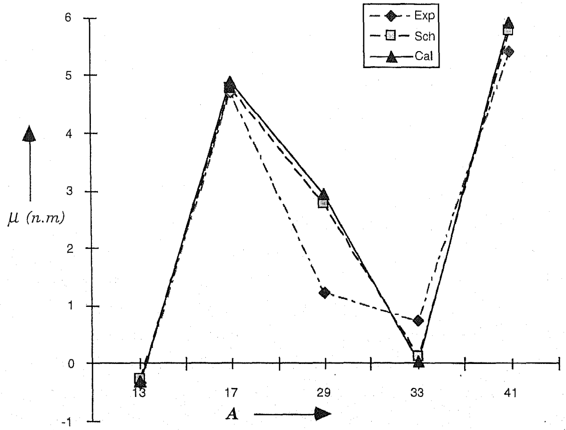


Figure V

Variation of magnetic moments (μ) of protron rich partner of the mirror nuclei pair with mass number (A). The plots are for experimental, Schmidt and calculated values in the present work.



are known as core polarization effects $\left(\delta\mu_{CP}\right)$. It is well known that the long range part of the NN interaction is transmitted by the one-pion exchange . The usual nuclear physics procedure for including these pionic degrees of freedom has been used to eliminate them explicitly and to express them emplicitly in terms of one-pion exchange potential (OPEP) which depends only on nucleon dynamical variables. The OPEP is an isospin-dependent charge exchange potential. In its presence the total nuclear current is not simply the sum of single nucleon current (impulse approximation) but must be accompanied by additional two-body current in order to satisfy charge conservation. Thus the pion (meson) exchange current describes the additional electromagnetic interaction of the external field with the underlined pion degrees of freedom and represents the pionic effect in the electromagnetic porperties of the nucleus. Such effects can also contribute to the magnetic moments and are referred to as $\delta\mu_{MEC}$ (*Hyuga et al,* 1980). If the nucleons are assigned quark degrees of freedom the effective electromagnetic currents in the nucleon space have additional non-local and isospin-dependent two body currents. These additional electromagnetic currents are referred to as "quark exchange currents" since they are a direct consequence of the underlined quark degrees of freedom and the antisymmetrisation principle on quark level. The correction due to the quark degrees of freedom on magnetic moments have been estimated by *Yamauchi et al* (1991). These authors have decomposed the effect of quark degrees of freedom on magnetic moments in two parts, that is, the one which generates the short-range part of the NN interaction and can be absorbed in the proper definition of NN interaction or NN wavefunction and the remaining part which cannot be described by that way. The former contribution is taken into account by considering short range corrections on the impulse current and is referred to as $\delta\mu_{SC}$ and the later is referred to as $\delta\mu_{QEC}$.

We list these correction for some of the nuclei where ever available in **Table 6.1-6**, along with correction due to six-quark cluster formation effects as estimated in the present work. As seen from the **Table 6.1-6** the quark cluster formation effects make a sizable contribution to the magnetic moments ranging from 3 to 15%. In almost all the nuclei the dominant correction comes from configuration mixing and the meson exchange current. In the case of ^{17}O a large amount of configuration mixing effect is canceled by the meson exchange current. The correction due to core polarisation, quark exchange current and short-range correlation effects are also significant. The correction due to quark bag formation effect is some what larger than the quark exchange current contribution, estimated by *Yamauchi et al.* (1991) but of the same sign. As explained in the text both these corrections arise due to quark degrees of freedom in nuclei but have been estimated following different approaches. In almost all the cases the estimated correction due to six-quark cluster formation is smaller than correction due to configuration mixing and meson exchange current effects. Since the quark bag formation takes place for small internuclear distance and one-pion correction are dominant for large distances, these two should be added in the estimation of magnetic moments. The correction due to mixing of configurations is caused due to intermingling of configurations of the nucleonic states and contributes over and above the correction due to quark degrees of freedom. Thus a comparison with the experimental data can be made if the quark effect corrections are combined with those arising due to other effects particularly due to configuration mixing and meson exchange current. If these corrections are added to the quark cluster correction, discrepancy with the experimental data decreases in some of the nuclei $\left({}^{13}_{6}C_7,\ {}^{13}_{7}N_6,\ {}^{29}_{14}Si_{15},\ {}^{41}_{20}Ca_{21},\ {}^{41}_{21}Sc_{20}\right)$. If the correction due to configuration mixing and meson exchange current are

Table 6.1-6

Correction to the Schmidt values of the magnetic moments as obtained in the present work and in various other calculations. $\delta\mu_{Cal}$: six quark cluster formation effect, $\delta\mu_{CM}$: configuration mixing effect, $\delta\mu_{CP}$: core polarization, $\delta\mu_{MEC}$: meson exchange current, $\delta\mu_{QEC}$: quark exchange current and $\delta\mu_{SC}$: short range correlation effect.

A	Nucleus	Present Work $\delta\mu_{Cal}$ (n.m.)	Other Calculations $\delta\mu_{CM}$ (n.m.)	$\delta\mu_{CP}$ (n.m.)	$\delta\mu_{MEC}$ (n.m.)	$\delta\mu_{QEC}$ (n.m.)	$\delta\mu_{SC}$ (n.m.)
13	$^{13}_{6}C_{7}$	0.031	—	0[c]	—	—	—
	$^{13}_{7}N_{6}$	-0.047	—	-0.016[c]	—	—	—
17	$^{17}_{8}O_{9}$	-0.072	0.28[b], 0.57[d]	—	-0.52[d]	-0.055[e]	-0.007[e]
	$^{17}_{9}F_{8}$	0.110	0.30[b], 0.75[d]	-	0.52[d]	0.055[e]	0.007[e]
29	$^{29}_{14}Si_{15}$	-0.104	1.37[a]	—	—	—	—
	$^{29}_{15}P_{14}$	0.156	—	—	—	—	—
33	$^{33}_{16}S_{17}$	0.062	—0.36[a]	—	—	—	—
	$^{33}_{17}Cl_{16}$	-0.093	—	—	—	—	—
41	$^{41}_{20}Ca_{21}$	-0.084	0.28[b]	—	—	—	—
	$^{41}_{21}Sc_{20}$	0.127	-0.28[b]	—	—	—	—

a) *Arima and Horie (1954).*

b) *Mavromatis and Zamick (1966).*

c) *Kitagawa (1999).*

d) *Arima et al (1987).*

e) *Yamauchi et al (1991).*

estimated in hybrid quark model framwork, a part of these contribution will be suppressed because of the cutting of nucleonic sector from 0 to r_0. It has been shown by *Radhey Shyam et al* (1988) that in the calculation of meson exchange current $\rho\pi\gamma$ term reduces by about 60% due to the large mass of ρ-meson when the contribution from the $r<r_0$ region is excluded. The pair current term is not suppressed as much. But the inclusion of this term is somewhat questionable since we know that the nucleons are having complex structure and $\pi N\overline{N}$ and $\gamma N\overline{N}$ couplings are expected to be strongly suppressed due to their off-shell nature.

The numerical result of our calculations show that the six-quark cluster formation effect makes a significant contribution to the magnetic moments of nuclei. Any comparison with the experimental data can only be warranted if these corrections are supplemented with the contribution due to configuration mixing and meson exchange current. One of the main drawbacks of such calculations is that the estimation of all these corrections are model dependent. However, we expect that the magnitude of quark bag formation correction would not change drastically because the six-quark probability has been calculated from the external region using the shell model wavefunctions. We have also ignored the non-central components in the nuclear wave functions. Moreover the assumption that the nuclear magnetic moment inside a bag increases by a factor of 4/3 compared to its free nucleon value is also model dependent (*Grande et al,* 1975 and *Tar and Donoghue*, 1983). Infact *Yamazaki* (1985) has observed from a carefull analysis of the proton-neutron asymmetric effect of the observed δg_l factor in the A = 208 region, that effective nuclear magneton in nuclei is only 8 ± 3% larger than the free value. We have also made calculations with $\mu_N' = 1.08\,\mu_N$ and the results are shown in **Table 6.1-7**.

Table 6.1-7

Calculated values of magnetic moments of mirror nuclei pairs with $\mu'_N = 1.08\,\mu_N$.

A	$P^{6q}_{NN}(r_0)^*$	Nucleus	μ'_{Cal} (n.m.)
13	0.151	${}^{13}_{6}C_{7}$	0.6442
		${}^{13}_{7}N_{6}$	-0.2746
17	0.117	${}^{17}_{8}O_{9}$	-1.9278
		${}^{17}_{9}F_{8}$	4.8161
29	0.168	${}^{29}_{14}Si_{15}$	-1.9357
		${}^{29}_{15}P_{14}$	2.8275
33	0.167	${}^{33}_{16}S_{17}$	1.1613
		${}^{33}_{17}Cl_{16}$	0.1036
41	0.137	${}^{41}_{20}Ca_{21}$	-1.9309
		${}^{41}_{21}Sc_{20}$	5.8206

* *Köch and Miller (1985).*

6.2 Deformed Bags

An alternative approch to account for the observed deviation from the Schmidt value is to consider the formation of deformed nucleonic bags inside nuclei. Earlier studies have shown (*Abbas* , 1987) that the formation of deformed nucleonic bags effect the magnetic properties of nuclei. We have estimated the probability of deformed bag formation $(P_D(A))$ from the experimental magnetic moments for nuclei with A = 13, 17, 29, 33, 41. The result are shown in **Table 6.2-1**. For free nucleon $P_D(N)$ is 1/4, but this value gets modifield inside nuclear medium. The European Muon Collaboration effect (*Close et al.* 1983; *Jaffe,* 1984; *Mathieu and Watson,* 1984 and *Celenza et al,* 1984) results indicate that there is an increase in the confinement size of the nucleons in the nuclear medium. If we accept this picture, then $P_D(A) < P_D(N)$. In a deformed bag there is a larger surface area in the flat region than at the edges. If the pull on the surface is uniform, greater expansion will take place perpendicular to the flat part and so the deformation will decrease with increasing A. This trend is not reflected in our results. The D-state probabilities are particularly large for A = 29, 33. These are also the nuclei for which the experimental magnetic moments differ largly from the Schmidt values. This is probably indicative of the fact that the simple picture of deformed bags is not suitable for these nuclei.

We have also calculated the magnitudes of the ratio of the deviation in the magnetic moments from the single nucleon values from equations (4.2-8), (4.2-9). The results are displayed as the ratio of the deviation for the mirror pair in **Table 6.2-2** and compared with the corresponding experimental values.

Table 6.2-1

Probability of deformed bag formation ($P_D(A)$) in light nuclei with A = 13, 17, 29, 33, 41.

A	Nucleus	$P_D(A)$
13	${}^{13}_{6}C_{7}$	0.1739
	${}^{13}_{7}N_{6}$	0.2000
17	${}^{17}_{8}O_{9}$	0.2576
	${}^{17}_{9}F_{8}$	0.2683
29	${}^{29}_{14}Si_{15}$	0.7832
	${}^{29}_{15}P_{14}$	0.6680
33	${}^{33}_{16}S_{17}$	0.5793
	${}^{33}_{17}Cl_{16}$	0.5306
41	${}^{41}_{20}Ca_{21}$	0.3747
	${}^{41}_{21}Sc_{20}$	0.3468

Table 6.2-2

Ratio of the deviation of deformed magnetic moments for the mirror pair A = 13, 17, 29, 33, 41. δ_{Exp} and δ_{Cal} are the experimental and calculated values of the deviation respectively.

A	Nucleus	δ_{Exp} *	δ_{Cal}
13	${}^{13}_{6}C_{7}$ ~ ${}^{13}_{7}N_{6}$	– 0.6915	–0.6726
17	${}^{17}_{8}O_{9}$ ~ ${}^{17}_{9}F_{8}$	–0.2838	–0.2750
29	${}^{29}_{14}Si_{15}$ ~ ${}^{29}_{15}P_{14}$	–0.8745	–0.8503
33	${}^{33}_{16}S_{17}$ ~ ${}^{33}_{17}Cl_{16}$	–0.6623	–0.6478
41	${}^{41}_{20}Ca_{21}$ ~ ${}^{41}_{21}Sc_{20}$	–0.8833	–0.8585

* *Experimental values of magnetic moments used in calculating δ_{Exp} are from Raghavan (1989).*

6.3 Hypernuclear Magnetic Moments

We have calculated the magnetic moments of few light hypernuclei with X–N–Λ configuration, where X denotes the nuclear core $\left({}^{4}He,\ {}^{12}C\right)$ and N a nucleon (n and p) respectively using equations (4.3-3) and (4.3-4). The experimental and the Schmidt values of magnetic moments of mirror nuclei pair A=5,13 are shown in **Table 6.3-1**. The predicted magnetic moments of ${}^{14}_{\Lambda}C^{*}$ and ${}^{14}_{\Lambda}N^{*}$ in **Table 6.3-1** are obtained from equations (4.3-1) and (4.3-2) using experimental and Schmidt values for the magnetic moments of the core nuclei. We have used experimental values of effective nuclear moments of core nuclei ${}^{13}C$ and ${}^{13}N$ to calculate magnetic moments of ${}^{14}_{\Lambda}C^{*}$ and ${}^{14}_{\Lambda}N^{*}$ hypernuclei for $J^{\pi} = 1^{-}$ state (the ground state is $J^{\pi} = 0^{-}$ state for these nuclei) from equations (4.3-3) and (4.3-4). In the case of ${}^{5}He$ and ${}^{5}Li$ as there are no experimental data, we have used Schmidt values in the formulae. The six-quark bag formation probabilities of a hyperon with the nucleons $\left(P^{6q}_{\Lambda N}(r_o)\right)$ needed as input in equations (4.3-3) and (4.3-4) have been calculated in both Moshinsky and Slater methods for different sets of oscillater length parameters shown in **Table 5-1**. The magnetic moment of a free hyperon is $\mu_{\Lambda} = -0.613$n.m. It is well known that because of its internal quark structure the baryonic properties differ in nuclei from those in isolation. Thus the effective magnetic moment of a hyperon μ'_{Λ} inside a six-quark hyperon-nucleon bag would be different than that of a free hyperon. Assuming that μ'_{Λ} increases in direct proportionalty to the size of the six-quark bag we have taken $\mu'_{\Lambda} = 4/3\ \mu_{\Lambda}$. The results of our calculation are shown in **Table 6.3-2** (Moshinsky method) and **Table 6.3-3** (Slater method) for ${}^{6}_{\Lambda}He$, ${}^{6}_{\Lambda}Li$, ${}^{14}_{\Lambda}C^{*}$ and ${}^{14}_{\Lambda}N^{*}$.

Table 6.3-1

Magnetic moments of light nuclei.

Nuclei	J^{π}	μ_{Exp} (n.m.)	μ_{Sch} (n.m.)
^{5}He	$\frac{3}{2}^{-}$	—	-1.913
^{5}Li	$\frac{3}{2}^{-}$	—	3.793
^{13}C	$\frac{1}{2}^{-}$	0.7024	0.6377
^{13}N	$\frac{1}{2}^{-}$	−0.3222	−0.2633
Hypernuclei	J^{π}	μ_{p} (n.m.)	μ_{Sch} (n.m.)
$^{6}_{\Lambda}He$	1^{-}	—	−1.288
$^{6}_{\Lambda}Li$	1^{-}	—	3.467
$^{14}_{\Lambda}C^{*}$	1^{-}	0.0894	0.0247
$^{14}_{\Lambda}N^{*}$	1^{-}	−0.9352	−0.8763

μ_p and μ_{Sch} are the predicted magnetic moments of A= 6, 14* mirror hypernuclei calculated from equations (4.3-1) and (4.3-2), using experimental and Schmidt values of nuclear moments μ_N respectively. A = 14* mirror hypernuclei $\left(^{14}_{\Lambda}C^{*},\ ^{14}_{\Lambda}N^{*} \right)$ are first excited states with $J^{\pi} = 1^{-}$.

Table 6.3-2

Magnetic moments of light hypernuclei with X–N–Λ configuration. Six-quark probabilities $P_{\Lambda N}^{6q}(r_0)$ are calculated in Moshinsky method with $r_0 = 1fm$ for different sets of oscillator length parameter **(Table 5-1).** *The effective magnetic moment of a hyperon (μ'_Λ) inside a hyperon-nucleon six-quark bag is taken to be $\frac{4}{3}\mu_\Lambda = 0.817 n.m.$*

Hyper-nuclei	J^π	Gal I		Mujib		Wang	
		$P_{\Lambda N}^{6q}(r_0)$	μ (n.m.)	$P_{\Lambda N}^{6q}(r_0)$	μ (n.m.)	$P_{\Lambda N}^{6q}(r_0)$	μ (n.m.)
${}^{6}_{\Lambda}He$	1^-	(0.0706)	-1.281	0.0396	-1.284	0.0503	-1.283
${}^{6}_{\Lambda}Li$	1^-	(0.0706)	3.475	0.0396	3.471	0.0503	3.472
${}^{14}_{\Lambda}C^*$	1^-	0.0973	0.070	0.0831	0.072	0.0889	0.071
${}^{14}_{\Lambda}N^*$	1^-	0.0973	-0.9550	0.0831	-0.952	0.0889	-0.953

Table 6.3-3

*Magnetic moment of light hypernuclei with X-N-Λ configuration. Six-quark probabilities $P_{\Lambda N}^{6q}(r_0)$ are calculated in Slater method with $r_0 = 1fm$ for different sets of oscillator length parameters **(Table5-1).** The effective magnetic moment of a hyperon (μ'_Λ) inside a hyperon-nucleon six-quark bag is taken to be $\frac{4}{3}\mu_\Lambda = 0.817 n.m.$*

Hyper-nuclei	**J^π**	**Gal II**		**Mujib**		**Wang**	
		$P_{\Lambda N}^{6q}(r_0)$	**μ (n.m.)**	**$P_{\Lambda N}^{6q}(r_0)$**	**μ (n.m.)**	**$P_{\Lambda N}^{6q}(r_0)$**	**μ (n.m.)**
$^{6}_{\Lambda}He$	1^-	0.0765	–1.280	0.0519	–1.282	0.0675	–1.281
$^{6}_{\Lambda}Li$	1^-	0.0765	3.475	0.0519	3.473	0.0675	3.474
$^{14}_{\Lambda}C^*$	1^-	0.1050	0.068	0.0711	0.075	0.1150	0.066
$^{14}_{\Lambda}N^*$	1^-	0.1050	–0.957	0.0711	–0.950	0.1150	–0.959

It is well known that quark exchange currents, that is the exchange currents arising from quark degrees of freedom, are one of the important effects which cause the change in the nucleonic properties inside the nuclei. *Takeuchi et al* (1988) has estimated the effect of quark exchange currents on the magnetic moments of nucleon-hyperon pairs. The magnetic moments of ΛN system without interaction (μ_{Sum}), the correction arising due to the quark exchange currents (μ_{QE}) and the corrected magnetic moment including the quark exchange corrections (μ_{Total}) are shown in **Table 6.3-4**. Using these values as input, the effective magnetic moment of Λ–hyperon in a Λn or Λp bag have been estimated from equations (4.3-5) and (4.3-6). The effect of quark exchange between Λ–particle and a spin closed shell vanishes. Thus in estimating the magnetic moment of hypernuclei using equations (4.3-3) and (4.3-4), the six-quark bag formation probability $P^{6q}_{\Lambda N}(r_0)$ is that of the hyperon with only the valence nucleon. The results obtained with quark exchange correction are shown in **Table 6.3-5**. The predicted magnetic moments of hypernuclei have been compared with those of other work for ${}^{6}_{\Lambda}He$ and ${}^{6}_{\Lambda}Li$ in **Table 6.3-6**. *Yamazaki* (1985) has shown that the magnetic moments for protons and neutrons in A = 208 region get enhanced by 8 ± 3%. Assuming SU(3) symmetry, we may expect a similar enhancement in the magnetic moment of Λ–particle. Thus we have also included in the table the results for the magnetic moments of hypernuclei with $\mu'_{\Lambda} = 1.08\,\mu_{\Lambda}$ (Set III). In the work of *Tanaka* (1989a, 1989b) first column gives the magnetic moments obtained using shell model wavefunctions with good isospin in the effective moment method (using equations 4.3-1 and 4.3-2), the values obtained after including the configuration mixing effects are shown in the second column. In the work of *Motoba et al* (1985) the magnetic moments of hypernuclei have been calculated in the microscopic three cluster method. The first column gives the magnetic moments when the clusters are treated as structureless particles. The values obtained after incorporating the internal structure of the cluster are shown in the second column.

Table 6.3-4

Magnitude of quark exchange effects on the magnetic moments of ΛN system in different spin configuration.

System →	Λp		Λn	
Spin→	S=1 ↑↑	S=0 ↑↓	S=1 ↑↑	S =0 ↑↓
μ_{Sum} (n.m.)	2.187	–3.413	–2.478	1.252
$\Delta\mu_{QE}$ (n.m)	-0.311	0.311	–0.311	0.311
μ_{Total} (n.m)	1.876	–3.102	–2.789	1.562

$\mu_{Sum}(n.m.)$: *Sum of the magnetic moment of a free nucleon and a free hyperon.*

$\Delta\mu_{QE}(n.m.)$: *Correction to the magnetic moment of ΛN system due to quark exchange effects.*

$\mu_{Total}(n.m.)$: *Sum of* μ_{Sum} *and* μ_{QE}

Table 6.3-5

Magnetic moment of light hypernuclei with X-N-Λ configuration including the effect of quark exchange current. Six-quark probabilities $P_{\Lambda N}^{6q}(r_0)$ are calculated in Slater method with $r_0 = 1fm$ The effective magnetic moments of Λ–particle (μ'_Λ) inside a hyperon-neutron and a hyperon-proton six-quark bag is –0.7445 n.m. and –0.7686 n.m. respectively.

Hyper-nuclei	J^π	Gal II		Mujib		Wang	
		$P_{\Lambda N}^{6q}(r_0)$	μ (n.m.)	$P_{\Lambda N}^{6q}(r_0)$	μ (n.m.)	$P_{\Lambda N}^{6q}(r_0)$	μ (n.m.)
${}^{6}_{\Lambda}He$	1^-	0.0285	–1.285	0.0171	–1.287	0.0221	–1.286
${}^{6}_{\Lambda}Li$	1^-	0.0285	3.470	0.0171	3.469	0.0221	3.469
${}^{14}_{\Lambda}C^*$	1^-	0.028	0.086	0.0204	0.087	0.029	0.085
${}^{14}_{\Lambda}N^*$	1^-	0.028	–0.939	0.0204	–0.938	0.029	–0 .940

Table 6.3-6

Comparison of predicted magnetic moments of A=6 hypernuclei with X–N–Λ configuration with other work.

μ (n.m)	$^{6}_{\Lambda}He$		$^{6}_{\Lambda}Li$	
	Present Work			
	Moshinsky Method	Slater Method	Moshinsky Method	Slater Method
I	−1.281	−1.280	3.474	3.475
II	−1.287	−1.286	3.468	3.469
III	−1.286	−1.286	3.469	3.470
	Other Work			
Motoba [a]	−1.155	−1.382	3.322	3.976
Tanaka [b]	−1.288	−1.301	3.467	3.461

I, II and III refer to following values of the effective hyperon magnetic moment for :

I : $\mu'_{\Lambda} = -0.817$ n.m.

II : $\mu'_{\Lambda} = -0.7445$ n.m. in Λn bag.

$= -0.7686$ n.m. in Λp bag.

III : $\mu'_{\Lambda} = -0.662$ n.m.

Calculated values are for Gal I parameters

a) *Motoba et al (1985).*

b) *Tanaka (1989a, 1989b).*

In the case of ${}^{6}_{\Lambda}He$ and ${}^{6}_{\Lambda}Li$ we observe that even a sizable variation in the six-quark probability for the formation of ΛN bag from ~2% to 8% makes a very small contribution towards the change in magnetic moments from the Schmidt values. Large variation in the values of μ'_{Λ} also make very little difference. This is not surprising in view of the fact that the magnetic moment of hyperon dependent terms contribute only ~16 % of the total magnetic moment of ${}^{6}_{\Lambda}He$ and ${}^{6}_{\Lambda}Li$. The ΛN bag formation effects increase the magnetic moments by a very small amount compared to the Schmidt values. *Tanaka* (1989a, 1989b) has estimated hyperon induced configuration mixing effect on magnetic moments of light hypernuclei described by the ΛN configuration. The estimated corrections are – 0.013n.m. for ${}^{6}_{\Lambda}He$ and 0.006n.m. for ${}^{6}_{\Lambda}Li$. If these contributions are added to our estimated values (Slater method I in **Table 6.3-6** which gives largest deviation from the Schmidt limits in the present work), the predicted magnetic moments of ${}^{6}_{\Lambda}He$ and ${}^{6}_{\Lambda}Li$ are given by –1.293n.m. and 3.481n.m. respectively. For both these nuclei the contribution from quark effects and configuration mixing effects are small and tend to cancel each other.

The level diagrams for ${}^{14}_{\Lambda}C^{*}$ and ${}^{14}_{\Lambda}N^{*}$ systems are shown in **Figure VI** and **Figure VII**, the ground state spin for both these nuclei is $J^{\pi} = 0^{-}$. In the case of ${}^{14}_{\Lambda}C^{*}$ the particle-instability threshold lies much higher at 5.4 ± 0.35MeV for the channel $\left(n+{}^{13}_{\Lambda}C\right)$. ${}^{14}_{\Lambda}C^{*}$ is predicted to have at least four particle- stable levels with several intruder states as shown in **Figure VI.** The lowest excited state is a 1^{-} state and decays into the 0^{-} level through M1 γ ray with a transition rate of 2.7×10^{12}sec^{-1}. ${}^{14}_{\Lambda}N^{*}$ has only one particle -stable excited state with $J^{\pi} = 1^{-}$, since the threshold energy for $\left(p+{}^{13}_{\Lambda}C\right)$ lies quite low, at 2.42 ± 0.35MeV. The 1^{-} state decays into 0^{-} state by emitting M1 γ ray with a transition rate of 1.7×10^{11}sec^{-1} as shown in **Figure VII.**

Figure VI

Energy level of ${}^{14}_{\Lambda}C$. *Energy values are shown towards the left and* J^{π} *values are shown towards the right.*

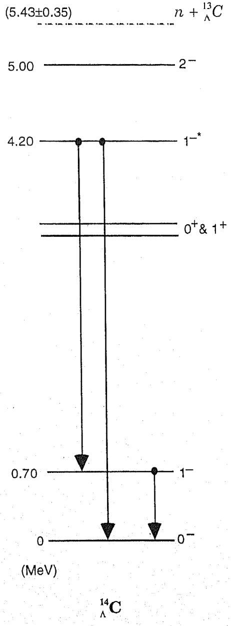
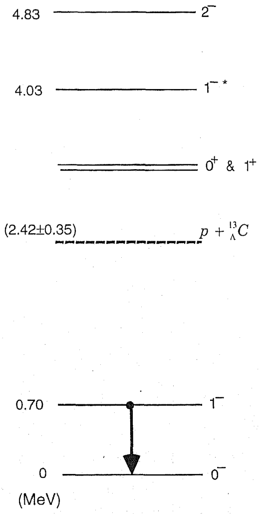


Figure VII

Energy level of ${}^{14}_{\Lambda}N$. *Energy values are shown towards the left and* J^{π} *values are shown towards the right.*



${}^{14}_{\Lambda}\mathbf{N}$

It is observed that the formation of six-quark ΛN bag makes a sizable change in the values of the magnetic moment with deviation from the Schmidt values ranging from 0.041n.m. to 0.062n.m. for ${}^{14}_{\Lambda}C^{*}$ and 0.083n.m. to 0.062n.m. for ${}^{14}_{\Lambda}N^{*}$. The corresponding deviation from the predicted values are 0.002n.m. to 0.023n.m. for ${}^{14}_{\Lambda}C^{*}$ and 0.003n.m. to 0.024n.m. for ${}^{14}_{\Lambda}N^{*}$. It is expected that the experimental moments of core nuclei include various contributions which cause the renormalization of the magnetic moments. Thus any departure from μ_p values can be a possible signature for the quark effects in the magnetic moments of hypernuclei. As the experimental values for magnetic moments of core nuclei ${}^{13}C$ and ${}^{13}N$ are well established the hypernuclei ${}^{14}_{\Lambda}C^{*}$ and ${}^{14}_{\Lambda}N^{*}$ are good candidates for the experimental observation of the magnetic moments.

We have also estimated the magnetic moments of hypernuclei with closed core+Λ configuration $\left({}^{5}_{\Lambda}He,\ {}^{13}_{\Lambda}C,\ {}^{17}_{\Lambda}O,\ {}^{41}_{\Lambda}Ca\right)$, using equations (4.3-9) and (4.3–10). $P^{6q}_{\Lambda N}(r_0)$ are taken to be the average probability for the formation of six-quark bag with the overlap of hyperon with various core nucleons. These probabilities have been calculated both in the Moshinsky and Slater method for different sets of oscillater length parameters (*Mujib et al,* 1979 and *Motoba et al,* 1994 for ${}^{5}_{\Lambda}He,\ {}^{13}_{\Lambda}C$; *Shlomo,* 1972 and *Madarres*, 1994 for ${}^{17}_{\Lambda}O,\ {}^{41}_{\Lambda}C)$ shown in **Table 6.3-7** . The predicted magnetic moments are given in **Table 6.3-8** (Moshinsky method) and **Table 6.3-9** (Slater method) both for $\mu'_{\Lambda} = \frac{4}{3}\mu_{\Lambda}$ and $\mu'_{\Lambda} = 1.08\mu_{\Lambda}$. The results obtained in the present work are compared with those of other authors in **Table 6.3-10.** It is observed that the magnetic moments of hypernuclei change by ~ 0.1 to 2.1%. The deviation being largest for ${}^{41}_{\Lambda}Ca$. *Cohen* and *Furnstahl* (1987) have

Table 6.3-7

Values of oscillater length parameters for nucleon (ν_N) *and hyperon* (ν_N) *used by different authors. The values of* ν_N *marked with* are obtained using relation* $\nu_\Lambda = (m_\Lambda / m_N)\nu_N$.

Hypernuclei	Set I		Set II	
	V_N *(fm⁻²)*	V_Λ *(fm⁻²)*	V_N *(fm⁻²)*	V_Λ *(fm⁻²)*
$^{5}_{\Lambda}He^{a}$	0.521	0.619*	0.532	0.632*
	0.521	0.336	0.521	0.457
$^{13}_{\Lambda}C^{a}$	0.359	0.426*	0.370	0.440*
	0.359	0.214	0.359	0.287
$^{17}_{\Lambda}O^{b}$	0.338	0.402*	0.397	0.472*
$^{41}_{\Lambda}Ca^{b}$	0.25	0.297*	0.26	0.309*

a) Mujib et al (1979) and Motoba et al (1994).

b) Shlomo (1972) and Madarres (1994).

Table 6.3-8

Magnetic moments of light hypernuclei with closed core+Λ configuration. Six-quark probabilities are calculated in Moshinsky method with $r_0 = 1fm$. The effective magnetic moment of a hyperon (μ'_Λ) inside a six-quark bag is taken to be $\frac{4}{3}\mu_\Lambda = -0.817\ n.m.$ The values marked with † are obtained with $\mu'_\Lambda = -1.08\,\mu_\Lambda = -0.662\ n.m.$

Hyper-nuclei	J^π	Set I		Set II	
		$P^{6q}_{\Lambda N}(r_0)$	μ (n.m.)	$P^{6q}_{\Lambda N}(r_0)$	μ (n.m.)
${}^{5}_{\Lambda}He^{a}$	$\frac{1}{2}^+$	0.0439	-0.621 -0.614†	0.0451	-0.622 -0.615†
${}^{13}_{\Lambda}C^{a}$	$\frac{1}{2}^+$	0.304	-0.619 -0.614†	0.0317	-0.619 -0.615†
${}^{17}_{\Lambda}O^{b}$	$\frac{1}{2}^+$	0.0497	-0.623 -0.615†	0.0624	-0.625 -0.616
${}^{41}_{\Lambda}Ca^{b}$	$\frac{1}{2}^+$	0.0701	-0.627 -0.616†	0.0725	-0.628 -0.617†

For ${}^{5}_{\Lambda}He$ and ${}^{13}_{\Lambda}C$ results are presented for oscillater length parameters obtained from 2nd and 4th rows of **Table 6.3-7.**

Table 6.3-9

Magnetic moments of light hypernuclei with closed core+Λ configuration. Six-quark probabilities are calculated in Slater method with $r_0 = 1fm$. The effective magnetic moment of a hyperon (μ'_Λ) inside a six-quark bag is taken to be $\frac{4}{3}\mu_\Lambda = -0.817$ n.m. The values marked with † are obtained with $\mu'_\Lambda = -1.08\,\mu_\Lambda = -0.662$ n.m.

Hyper-nuclei	J^π	Set I		Set II	
		$P^{6q}_{\Lambda N}(r_0)$	μ (n.m.)	$P^{6q}_{\Lambda N}(r_0)$	μ (n.m.)
${}^{5}_{\Lambda}He^{a}$	$\frac{1}{2}^+$	0.0559	-0.624 -0.616†	0.0715	-0.628 -0.617†
${}^{13}_{\Lambda}C^{a}$	$\frac{1}{2}^+$	0.0513	-0.624 -0.616†	0.0630	-0.626 -0.616†
${}^{17}_{\Lambda}O^{b}$	$\frac{1}{2}^+$	0.0720	-0.628 -0.617†	0.0903	-0.632 -0.617†
${}^{41}_{\Lambda}Ca^{b}$	$\frac{1}{2}^+$	0.0990	-0.633 -0.618†	0.1049	-0.634 -0.618†

*All results are presented for oscillater length parameters marked with $*$ in 1st, 3rd, 5th and 6th rows of **Table 6.3-7**.*

Table 6.3-10

Comparison of predicted magnetic moments of hypernuclei with closed core+Λ hypernuclei, with those of other calculations.

Reference	μ (n.m.)			
	${}^{5}_{\Lambda}He$	${}^{13}_{\Lambda}C$	${}^{17}_{\Lambda}O$	${}^{41}_{\Lambda}Ca$
Present Work				
MMI	**–0.621**	**–0.619**	**–0.623**	**–0.627**
MM2	**–0.614**	**–0.614**	**–0.615**	**–0.616**
SMI	**–0.624**	**–0.624**	**–0.628**	**–0.633**
SM2	**–0.616**	**–0.616**	**–0.617**	**–0.618**
Other Work				
Cohen[a]	—	**–0.650**	**–0.648**	**–0.665**
Mores[b]	—	**–0.649**	**–0.643**	**–0.656**
Gattone[c]	—	**–0.611**	**–0.611**	**-0611**
Schmidt	—	**–0.613**	**–0.613**	**–0.613**

MM1 : *Moshinsky method* ($\mu'_\Lambda = {}^4\!/_3\,\mu_\Lambda$).

MM2 : *Moshinsky method* ($\mu'_\Lambda = 1.08\mu_\Lambda$).

SM1 : *Slater method* ($\mu'_\Lambda = {}^4\!/_3\,\mu_\Lambda$).

SM2 : *Slater method* ($\mu'_\Lambda = 1.08\mu_\Lambda$).

a) Cohen and Furnstahl (1987).

b) Mares and Zofka (1990).

c) Gattone et al (1991).

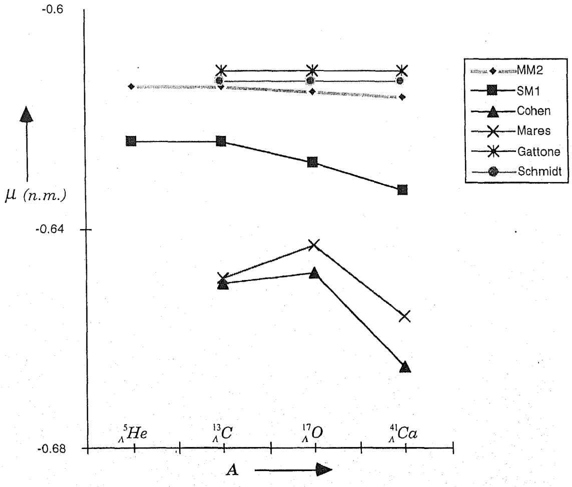
In the present work result are tabulated for parameters of Set I with cut off radius r_0 = 1fm.

obtained the magnetic moments in a self-consistant calculations by evaluating the effect of the linear core response to the hyperon in the local density approximation. *Mares* and Zofka (1990) have made self-consistant calculations of the entire core+Λ system and incorporated this effect automatically. Both these calculations have obtained remarkable deviation from the Schmidt values. *Gattone et al* (1991) has estimated the magnetic moments in the relativistic meanfield model including a tenser term to describe the $\sigma - \omega$ coupling and have obtained reults very close to standard Schmidt value even in the presence of strong renormalization current. In **Figure VIII** we show the magnetic moments of ${}^{5}_{\Lambda}He$, ${}^{13}_{\Lambda}C$, ${}^{17}_{\Lambda}O$, and ${}^{41}_{\Lambda}Ca$, obtained in the present work, other calculations and the Schmidt value. Again, if various renormalization processes tend to cancel each other, any departure from the Schmidt limit in the observed magnetic moments can be attributed to the quark effects.

To the best of our knowledge no measurment of hypernuclear magnetic moments have been performed as yet, although some are planned at KEK in Japan (*Fukuda et al,* 1986). In Hypernuclei hyperon carry non zero strangeness which distinguishes them from nucleons. Thus they are hardly or not Pauli excluded from orbitals occupied by nucleons and can penetrate dense nuclear matter inaccessible to other hadronic probes. We can use hypernuclei to explore the role of hyperons in hadronic forces and to measure changes of electromagnetic properties of hyperons by the nuclear environment. Thus the experimental data on the measurement of magnetic moment of hypernuclei is much awaited.

Figure VIII

Plot of magnetic moments (μ) of hypernuclei with closed core+Λ configuration, with mass number (A).



APPENDIX

Appendix A

Moshinsky and Smirnov Coefficients

Moshinsky Coefficient: Consider the hamiltonian of the two nucleons in a harmonic oscillator potential of frequency w as

$$H(1,2) = \frac{1}{2}\left(p_1^2/m\right) + \frac{1}{2}\,mw^2r_1^2 + \frac{1}{2}\left(p_2^2/m\right) + \frac{1}{2}\,mw^2r_2^2 \tag{A.1}$$

where $\boldsymbol{r}_1, \boldsymbol{r}_2$ and $\boldsymbol{p}_1, \boldsymbol{p}_2$ are the coordinates and momenta of the two nucleons and m their mass. The relative coordinate $\boldsymbol{r}$ and the centre of mass coordinate $\boldsymbol{R}$ may be defined as,

$$\boldsymbol{r} = \frac{1}{\sqrt{2}}\left(\boldsymbol{r}_1 - \boldsymbol{r}_2\right)$$

$$\boldsymbol{R} = \frac{1}{\sqrt{2}}\left(\boldsymbol{r}_1 + \boldsymbol{r}_2\right) \tag{A.2}$$

The corresponding momenta are

$$\boldsymbol{p} = \frac{1}{\sqrt{2}}\left(\boldsymbol{p}_1 - \boldsymbol{p}_2\right)$$

$$\boldsymbol{P} = \frac{1}{\sqrt{2}}\left(\boldsymbol{p}_1 + \boldsymbol{p}_2\right) \tag{A.3}$$

The hamiltonian can be written as,

$$H(1,2) = \frac{1}{2}\left(p^2/m\right) + \frac{1}{2}\,mw^2r^2 + \frac{1}{2}\left(P^2/m\right) + \frac{1}{2}\,mw^2R^2 \tag{A.4}$$

The angular momenta associated with the different coordinates will be designated by l_1, l_2, l, L. From the conservation of angular momentum

$$\boldsymbol{l}_1+\boldsymbol{l}_2=\lambda=\boldsymbol{l}+\boldsymbol{L} \tag{A.5}$$

Since the expressions (A.1) and (A.4) describe the same hamiltonian $H(1,2)$, both $\phi_{n_1l_1m_1}(\boldsymbol{r}_1)\,\phi_{n_2l_2m_2}(\boldsymbol{r}_2)$ and $\phi_{NLM}(\boldsymbol{R})\,\phi_{nlm}(\boldsymbol{r})$form a complete set of wavefunction of two particles moving in the harmonic oscillator potential. Thus any one product wavefunction $\phi_{n_1l_1m_1}(\boldsymbol{r}_1)\,\phi_{n_2l_2m_2}(\boldsymbol{r}_2)$ should be expressed in terms of a complete set of harmonic oscillator functions $\phi_{NLM}(\boldsymbol{R})\,\phi_{nlm}(\boldsymbol{r})$. The wave function for a single harmonic oscillator will be given by

$$R_{nl}(r)Y_{lm}(\theta,\varphi) \tag{A.6}$$

where Y_{lm} are spherical harmonics and $R_{nl}(r)$the radial functions. As λ commutes with the hamiltonian, the angular momentum coupled wavefunctions are defined as

$$\begin{aligned}&|n_1l_1,n_2l_2,\lambda\mu\rangle\\&=\sum_{m_1m_2}\langle l_1l_2m_1m_2|\lambda\mu\rangle R_{n_1l_1}(r_1)Y_{l_1m_1}(\theta_1,\varphi_1)R_{n_2l_2}(r_2)Y_{l_2m_2}(\theta_2,\varphi_2),\end{aligned} \tag{A.7}$$

$$|nl,NL,\lambda\mu\rangle=\sum_{mM}\langle lLmM|\lambda\mu\rangle R_{nl}(r)Y_{lm}(\theta,\varphi)R_{NL}(R)Y_{LM}(\Theta,\phi), \tag{A.8}$$

where $\langle l_1l_2,m_1m_2|\lambda\mu\rangle$, etc., are Clebsch-Gordan coefficients. Connecting equations (A.7) and (A.8),

$$|n_1l_1,n_2l_2,\lambda\mu\rangle=\sum_{nlNL}|nlNL;\lambda\mu\rangle\langle nlNL;\lambda|n_1l_1n_2l_2;\lambda\rangle_{MB} \tag{A.9}$$

where the quantity denoted with $\langle nlNL;\lambda|n_1l_1n_2l_2;\lambda\rangle_{MB}$ are Moshinsky transformation brackets. Because of conservation of energy and parity, these brackets will only be different from zero if the following relations are satisfied,

$$2n_1+l_1+2n_2+l_2=2n+l+2N+L, \tag{A.10}$$

$$(-1)^{l_1+l_2}=(-1)^{l+L} \tag{A.11}$$

These brackets are independent of magnetic quantum number μ. These brackets have been tabulated by *Moshinsky* (1959) and *Brody et al* (1960).

The matrix element of two body nucleon state in *j-j* coupling $|n_1l_1j_1,n_2l_2j_2;J\rangle$ can be transformed to LS coupling using A- transformation

$$|n_1l_1j_1,n_2l_2j_2;J\rangle=\sum_{\lambda S}A\begin{Bmatrix} l_1 & \frac{1}{2} & j_1 \\ l_2 & \frac{1}{2} & j_2 \\ \lambda & S & J \end{Bmatrix}|n_1l_1n_2l_2;\lambda S;J\rangle \tag{A.12}$$

The LS coupled wavefunctions can be changed over to relative and centre of mass representation using Moshinsky transformation brackets. Thus equation (A.12) becomes

$$|n_1l_1j_1,n_2l_2j_2;J\rangle=\sum_{\substack{\lambda S\\ nlNL}}A\begin{Bmatrix} l_1 & \frac{1}{2} & j_1 \\ l_2 & \frac{1}{2} & j_2 \\ \lambda & S & J \end{Bmatrix}\langle nlNL;\lambda|n_1l_1n_2l_2;\lambda\rangle\ |nlNL;\lambda S;J\rangle \tag{A.13}$$

The matrix element of the central interaction in *j-j* coupling can be written as

$$\langle n_1l_1j_1,n_2l_2j_2;J|V\ (\boldsymbol{r}_1-\boldsymbol{r}_2)|n_1l_1j_1,n_2l_2j_2;J\rangle$$

$$\begin{aligned} =\ &\sum_{\substack{\lambda S\\ \lambda' S'\\ nlNL\\ n'l'N'L'}}A\begin{Bmatrix} l_1 & \frac{1}{2} & j_1 \\ l_2 & \frac{1}{2} & j_2 \\ \lambda & S & J \end{Bmatrix}A\begin{Bmatrix} l_1 & \frac{1}{2} & j_1 \\ l_2 & \frac{1}{2} & j_2 \\ \lambda' & S' & J \end{Bmatrix}\langle nlNL;\lambda|n_1l_1n_2l_2;\lambda\rangle \\ &\times\langle n'l'N'L';\lambda'|n_1l_1n_2l_2;\lambda'\rangle \\ &\times\delta_{nn'}\delta_{NN'}\delta_{LL'}\delta_{ll'}\delta_{\lambda\lambda'}\ \langle nl\|V\ (\boldsymbol{r}_1-\boldsymbol{r}_2)\|nl\rangle \end{aligned} \tag{A.14}$$

If the interaction is conserved to angular momentum and parity, then its spin dependent equation (A.14) reduces to

$$\langle n_1 l_1 j_1, n_2 l_2 j_2; J | V(\boldsymbol{r}_1 - \boldsymbol{r}_2) | n_1 l_1 j_1, n_2 l_2 j_2; J \rangle$$

$$= \sum_{\substack{\lambda S \\ nlNL}} A^2 \begin{Bmatrix} l_1 & \frac{1}{2} & j_1 \\ l_2 & \frac{1}{2} & j_2 \\ \lambda & S & J \end{Bmatrix} \left[\langle nlNL; \lambda | n_1 l_1 n_2 l_2; \lambda \rangle \right]^2 \langle nl \| V(\boldsymbol{r}_1 - \boldsymbol{r}_2) \| nl \rangle \qquad \text{(A.15)}$$

Smirnov Coefficient : Consider two brayon of different mass moving in a harmonic oscillater potential of frequency w , the hamiltonian can be expressed as

$$H'(1,2) = \frac{1}{2}\left(p_1^2/m_1\right) + \frac{1}{2} m_1 w^2 r_1^2 + \frac{1}{2}\left(p_2^2/m_2\right) + \frac{1}{2} m_2 w^2 r_2^2 \qquad \text{(A.16)}$$

The eigenfunction of the Hamiltonian are of the form

$$\phi_{n_1 l_1 m_1}(\boldsymbol{r}_1, \boldsymbol{r}_{01})\, \phi_{n_2 l_2 m_2}(\boldsymbol{r}_2, \boldsymbol{r}_{02}) \qquad \text{(A.17)}$$

where

$$r_{01} = \left(\frac{\hbar}{m_1 w}\right)^{\frac{1}{2}},$$

$$r_{02} = \left(\frac{\hbar}{m_2 w}\right)^{\frac{1}{2}} \qquad \text{(A.18)}$$

In passing to the relative and centre-of-mass system

$$\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2,$$

$$\boldsymbol{R} = \frac{m_1 \boldsymbol{r}_1 + m_2 \boldsymbol{r}_2}{m_1 + m_2} \qquad \text{(A.19)}$$

The hamiltonian (A.16) assumes the form

$$H'(1,2)=\frac{1}{2}\left(p^2/\overline{m}\right)+\frac{1}{2}\overline{m}w^2r^2+\frac{1}{2}\left(P^2/M\right)+\frac{1}{2}Mw^2R^2 \tag{A.20}$$

where

$$\overline{m}=\frac{m_1m_2}{m_1+m_2},$$

$$M=m_1+m_2 \tag{A.21}$$

The eigenfunctions products of equation (A.20) are

$$\phi_{nlm}\left(\boldsymbol{r},\boldsymbol{r}_0\right)\,\phi_{NLM}\left(\boldsymbol{R},\boldsymbol{R}_0\right) \tag{A.22}$$

where

$$r_0=\left(\frac{\hbar}{mw}\right)^{1/2},$$

$$R_0=\left(\frac{\hbar}{Mw}\right)^{1/2} \tag{A.23}$$

The transformation connecting the angular momentum coupled wave function can be expressed in the form

$$\left|n_1l_1,n_2l_2;\lambda\mu\right\rangle_{r_{01}r_{02}}=\sum_{nlNL}\left|nl,NL,\lambda\mu\right\rangle\left\langle nlNL;\lambda\middle|n_1l_1n_2l_2;\lambda\right\rangle_{SM} \tag{A.24}$$

where

$$\left|n_1l_1,n_2l_2,\lambda\mu\right\rangle_{r_{01}r_{02}}=\phi_{n_1l_1m_1}\left(\boldsymbol{r}_1,\boldsymbol{r}_{01}\right)\phi_{n_2l_2m_2}\left(\boldsymbol{r}_2,\boldsymbol{r}_{02}\right),$$

$$\left|nl,NL,\lambda\mu\right\rangle_{r_0R_0}=\phi_{nlm}\left(\boldsymbol{r},\boldsymbol{r}_0\right)\phi_{NLM}\;\left(\boldsymbol{R},\boldsymbol{R}_0\right) \tag{A.25}$$

The quantity denoted with $\langle nlNL;\lambda\,|n_1l_1n_2l_2;\lambda\rangle_{SM}$ are Smirnov coefficients (*Smirnov,* 1961,1962 and *Bakri,* 1967). These coefficients can be expressed as

$$\langle nlNL;\lambda|n_1l_1n_2l_2;\lambda\rangle_{SM}=(-1)^{n_1+n_2+N+n}2^{\frac{1}{2}(n_1+n_2+l_1+l_2-N-n-L-l)}$$

$$\times\left[\frac{N!n!(2L+2N+1)!!(2l+2n+1)!!}{n_1!n_2!(2l_1+2n_1+1)!!\,(2l_2+2n_2+1)!!}\right]^{\frac{1}{2}}\frac{R_0^{2N+L}r_0^{2n+l}}{r_{01}^{2n_1+l_1}r_{02}^{2n_2+l_2}}$$

$$\times\sum(-1)^{s_3+f_2}\frac{n_1!n_2!}{s_1!s_2!s_3!q_1!q_2!q_3!}2^{s_3+q_3}$$

$$\times\left[\binom{2l_1}{2F_1}\binom{2l_2}{2F_2}(2L_1+1)(2L_2+1)(2J+1)(2\lambda+1)\right]^{\frac{1}{2}}$$

$$\times\,C^{L_1 0}_{F_1 0 F_2 0}\,C^{L_2 0}_{f_1 0 f_2 0}\,C^{L0}_{J0L_1 0}\,C^{l0}_{J0L_2 0}(2l_1+1)(2l_2+1)$$

$$\times G(s_3+q_3,J)a^{f_1+2s_2+s_3}b^{f_2+2q_2+q_3}\begin{Bmatrix}F_1 & f_1 & l_1\\ F_2 & f_2 & l_2\\ L_1 & L_2 & \lambda\end{Bmatrix}\begin{Bmatrix}J & J & 0\\ L_1 & L_2 & \lambda\\ L & l & \lambda\end{Bmatrix}. \qquad \text{(A.26)}$$

Summation in equation (A.26) is performed over all values of the subscripts $F_1F_2f_1f_2s_1s_2s_3q_1q_2q_3L_1L_2J$ with the following restrictions :

$$F_1+f_1=l_1,\qquad F_1=0,1,2.........l_1,$$

$$F_2+f_2=l_2,\qquad F_2=0,1,2.........l_2,$$

$$s_1+s_1+s_3=n_1,\quad s_i\geq 0,\quad \text{(all } s_i \text{ are integers)},$$

$$q_1+q_2+q_3=n_2,\quad q_i\geq 0,\quad \text{(all } q_i \text{ are integers)},$$

$$F_1+F_2+2s_1+2q_1+s_3+q_3=2N+L,$$

$$f_1+f_2+2s_2+2q_2+s_3+q_3=2n+l,$$

$$J=s_3+q_3,\,s_3+q_3-2,......,1\ or\ 0. \qquad \text{(A.27)}$$

The momenta L_1, L_2 assume any values permitted by the triangular rules in vectorial addition.

$$\boldsymbol{F}_1 + \boldsymbol{F}_2 = \boldsymbol{L}_1,$$

$$\boldsymbol{f}_1 + \boldsymbol{f}_2 = \boldsymbol{L}_2,$$

$$\boldsymbol{J} + \boldsymbol{L}_1 = \boldsymbol{L},$$

$$\boldsymbol{J} + \boldsymbol{L}_2 = \boldsymbol{l},$$

$$\boldsymbol{L}_1 + \boldsymbol{L}_2 = \lambda \tag{A.28}$$

In equation (A.26) $a = (r_{01}/r_0)^2$ and $b = (r_{02}/r_0)^2$.

Appendix B

Slater Integral

The direct and the exchange integral of potential $V(\boldsymbol{r}_1 - \boldsymbol{r}_2)$ can be solved by a technique developed (*Slater* , 1929) in atomic spectroscopy. The procedure is to expand $V(\boldsymbol{r}_1 - \boldsymbol{r}_2)$ in a series of Legendre polynomials, the argument of which is the angle θ between $\boldsymbol{r}_1$and $\boldsymbol{r}_2$ with coefficients which are functions of $|\boldsymbol{r}_1|$ and $|\boldsymbol{r}_2|$.

$$V(\boldsymbol{r}_1 - \boldsymbol{r}_2) = \sum_{\lambda=0}^{\infty} \vartheta_\lambda(r_1, r_2) P_\lambda(cos\,\theta) \tag{B.1}$$

$P_\lambda(\cos\theta)$ can be expanded in a finite set of spherical harmonics which are functions of $(\theta_1\varphi_1)$ and $(\theta_2\ \varphi_2)$ repectively and where

$$\vartheta_\lambda(r_1, r_2) = \tfrac{2\lambda+1}{2} \int_{-1}^{+1} V(\boldsymbol{r}_1 - \boldsymbol{r}_2) P_\lambda(\cos\theta) d(\cos\theta) \tag{B.2}$$

The integrals can be expressed as the products of radial and angular parts. Angular part can be integrated by using the standard techniques of angular momentum algebra. The radial integrals or Slater integrals F_λ in the direct matrix element are given by

$$F_\lambda = \int \left| \tfrac{R_{n_1 l_1}(r_1)}{r_1} \ \tfrac{R_{n_2 l_2}(r_2)}{r_2} \right|^2 \vartheta_\lambda(r_1, r_2) r_1^2 dr_1 r_2^2 dr_2 \tag{B.3}$$

Similarly the radial integrals in the exchange matrix element are

$$G_\lambda = \int \frac{R_{n_1l_1}(r_1)}{r_1}\frac{R_{n_2l_2}(r_1)}{r_1}\frac{R_{n_1l_1}(r_2)}{r_2}\frac{R_{n_2l_2}(r_2)}{r_2}\vartheta_\lambda(r_1,r_2)r_1^2dr_1r_2^2dr_2 \tag{B.4}$$

If the harmonic oscillater wave function are used to describe the nucleons, the product of wavefunctions $\phi_{n_1l_1m_1}(\boldsymbol{r}_1)\ \phi_{n_2l_2m_2}(\boldsymbol{r}_2)$ can be transformed to a sum of products $\phi_{NLM},(\boldsymbol{R})\,\phi_{nlm}(\boldsymbol{r})$, with the following restrictions, (i) $2n_1+l_1+2n_2+l_2=2n+l+2N+L$ for energy conservation, (ii)$l_1+l_2=\lambda=l+L$ for angular momentum conservation. $\boldsymbol{R}$ and $\boldsymbol{r}$ refer to the center of mass and relative coordinates of two nucleons and are defined as

$$\begin{aligned}\boldsymbol{r} &= \boldsymbol{r}_2-\boldsymbol{r}_1\\ 2\boldsymbol{R} &= \boldsymbol{r}_2+\boldsymbol{r}_1\end{aligned} \tag{B.5}$$

where

$$r_1^2 = R^2 + r^2\!/\!_4 - Rr\cos\alpha,$$

$$r_2^2 = R^2 + r^2\!/\!_4 + Rr\cos\alpha, \tag{B.6}$$

$$r_1\cdot r_2 = r_1r_2\cos\theta = \frac{1}{4}\left(4R^2-r^2\right), \tag{B.7}$$

$$r_1^2+r_2^2 = \frac{1}{2}\left(4R^2+r^2\right) \tag{B.8}$$

The volume element transforms to

$$r_1^2dr_1r_2^2dr_2d(cos\,\theta) = R^2dR\ r^2dr\ d(cos\,\theta) \tag{B.9}$$

α is the angle between vectors $\boldsymbol{R}$ and $\boldsymbol{r}$. The radial harmonic oscillator wavefunction is of the form

$$R_{nl}(r) = N_{nl}\ e^{-\frac{v}{2}r^2}r^{l+1}\vartheta_{nl}(r) \tag{B.10}$$

where N_{nl} is normalization factor, $\vartheta_{nl}(r)$ is associated Laguerre polynomial and v is the oscillator length parameter

$$\vartheta_{nl}(r) = L_{n+l+\frac{1}{2}}^{l-\frac{1}{2}}\left(vr^2\right), \tag{B.11}$$

$$N_{nl}^2 = \frac{2^{l-n+2}(2l+2n+1)!!\ v^{l+\frac{3}{2}}}{\sqrt{\pi}\quad n!\left[(2l+1)!!\right]^2} \tag{B.12}$$

Using transformation by equations (B.5)-(B.12) the direct and the exchange integral in equations (2.2B-3) – (2.2B-5) can be changed in terms of R and r.

The formulae for the six-quark probability density for the mirror hypernuclei pair ${}^{6}_{\Lambda}He \sim {}^{6}_{\Lambda}Li$ and ${}^{14}_{\Lambda}C \sim {}^{14}_{\Lambda}N$ are given below.

${}^{6}_{\Lambda}He \sim {}^{6}_{\Lambda}Li$: In ${}^{6}_{\Lambda}He \sim {}^{6}_{\Lambda}Li$ the overlap probability is determined between $0s_{\frac{1}{2}}$ core nucleon and $0p_{\frac{1}{2}}$ valence nucleon. The direct (equation 2.2b-3) and the exchange term (equation 2.2b-4) in the overlap probability reduce to

$$P^{d}_{n_i l_i j_i}(r_0) = C_0\left[2(B1)\,T(1) + \left(\frac{A1}{2}\right)T(2)\right], \tag{B.13}$$

$$P^{e}_{n_i l_i j_i}(r_0) = \frac{C_0}{2}\left[2(B1)\,T(1) - \left(\frac{A1}{2}\right)T(2)\right], \tag{B.14}$$

where

$$C_0 = \left(\frac{16}{3}\right)\left(\frac{v_N^4}{\pi}\right),$$

$$A1 = \int_0^{\infty} R^2 e^{-2v_N R^2}\, dR,$$

$$B1 = \int_0^{\infty} R^4 e^{-2v_N R^2}\, dR,$$

$$T(1) = \int_0^{r_0} r^2 e^{-\nu_N r^2/2} dr,$$

$$T(2) = \int_0^{r_0} r^4 \, e^{-\nu_N r^2/2} dr \tag{B.15}$$

$^{14}_{\Lambda}C \sim ^{14}_{\Lambda}N$: In $^{14}_{\Lambda}C \sim ^{14}_{\Lambda}N$ pair the six-quark formation probability for the valence nucleon can be expressed as the sum of the probability of the overlap of the $0s_{1/2}$ core nucleon and with $0p_{3/2}$ core nucleon.

$$P^{6q}_{NN}(r_0) = {}^{I}P^{6q}_{NN}(r_0) + {}^{II}P^{6q}_{NN}(r_0) \tag{B.16}$$

${}^{I}P^{6q}_{NN}(r_0)$ is given by expression similar to equation (B.13) and (B.14) in $^{6}_{\Lambda}He \sim ^{6}_{\Lambda}Li$. ${}^{II}P^{6q}_{NN}(r_0)$ gets reduced to the following expressions

$${}^{II}P^{d}_{n_i l_i j_i}(r_0) = C_{00}\left[2(C1)T(1) + \left(\frac{A1}{4}\right)T(3) + \left(\frac{B1}{3}\right)T(2)\right], \tag{B.17}$$

$${}^{II}P^{e}_{n_i l_i j_i}(r_0) = \frac{5C_{00}}{16}\left[16(C1)T(1) + (A1)\,T(3) - \left(\frac{40}{3}\right)(B1)\,T(2)\right], \tag{B.18}$$

where

$$C_{00} = \left(\frac{32}{9}\right)\left(\frac{\nu_N^5}{\pi}\right),$$

$$C1 = \int_0^{\infty} R^6 e^{-2\nu_N R^2} dR,$$

$$T(3) = \int_0^{r_0} r^6 e^{-\nu_N r^2/2} dr \tag{B.19}$$

In evaluation of Coulomb energy difference given by equations (3.3-4), (3.3-5) and (3.3-6), the direct and exchange terms are solved in a similar manner for mirror pairs $^{6}_{\Lambda}He \sim ^{6}_{\Lambda}Li$ and $^{14}_{\Lambda}C \sim ^{14}_{\Lambda}N$.

$^{6}_{\Lambda}\boldsymbol{He} \sim ^{6}_{\Lambda}\boldsymbol{Li}$: In $^{6}_{\Lambda}He \sim ^{6}_{\Lambda}Li$ pair the Coulomb energy difference in terms of direct and exchange terms are written as

$$\Delta_D = C_0\left[2(B1)TT(1) + \left(\frac{A1}{2}\right)TT(2)\right], \tag{B.20}$$

$$\Delta_{exch} = \frac{C_0}{2}\left[2(B1)TT(1) - \left(\frac{A1}{2}\right)TT(2)\right] \tag{B.21}$$

where

$$TT(1) = \int_0^{r_0} r\, e^{-\nu_N r^2/2} dr,$$

$$TT(2) = \int_0^{r_0} r^3 e^{-\nu_N r^2/2} dr \tag{B.22}$$

$^{14}_{\Lambda}\boldsymbol{C} \sim ^{14}_{\Lambda}\boldsymbol{N}$: In $^{14}_{\Lambda}C \sim ^{14}_{\Lambda}N$ pair the Coulomb energy difference for the valence nucleons can be expressed as the sum of the Coulomb energy of the overlap of the $0s_{1/2}$ core nucleon and with $0p_{1/2}$ core nucleon.

$$V_c = {}^{I}V_c + {}^{II}V_c \tag{B.23}$$

In ${}^{I}V_c$ the expression for the direct and exchange terms are similar to equation (B.22), (B.23) and (B.24). The expression for ${}^{II}V_c$ is given as,

$${}^{II}\Delta_D = C_{00}\left[2(C1)TT(1) + \left(\frac{A1}{4}\right)TT(3) + \left(\frac{B1}{3}\right)TT(2)\right], \tag{B.24}$$

$${}^{II}\Delta_{exch} = \frac{5C_{00}}{16}\left[16(C1)\, TT(1) + (A1)TT(3) - \left(\frac{40}{3}\right)(B1)TT(2)\right] \tag{B.25}$$

where

$$TT(3) = \int_0^{r_0} r^5 e^{-\vartheta_N r^2/2} dr \tag{B.26}$$

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SIX QUARK CLUSTER EFFECTS AND Λ-BINDING ENERGY DIFFERENCE BETWEEN $A=6$ MIRROR HYPERNUCLEI ${}^{6}_{\Lambda}\mathrm{He}$–${}^{6}_{\Lambda}\mathrm{Li}$

INDIRA MEHROTRA AND MADHULIKA MEHROTRA

Department of Physics, Allahabad University
Allahabad-211 002, India

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The contribution of the six quark cluster formation of the overlapping nucleons to the Λ-binding energy difference between the mirror hypernuclei pair ${}^{6}_{\Lambda}\mathrm{He}$–${}^{6}_{\Lambda}\mathrm{Li}$ has been estimated in the hybrid quark nucleon model. The contribution is small and model dependent. It makes the neutron rich nucleus ${}^{6}_{\Lambda}\mathrm{He}$ more bound compared to its proton rich partner ${}^{6}_{\Lambda}\mathrm{Li}$.

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1. Introduction

The study of binding energy difference between mirror nuclei has been a subject of interest since long [1–4]. With the availability of reliable experimentalal data on Λ-binding energies such studies have also been extended to mirror hypernuclei [5, 6]. The most important comtribution to this binding energy difference of mirror pair originates from the Coulomb energy, but as is well known the Coulomb energy contribution is not sufficient to explain the observed difference in the energy. The discrepancy which is non Coulombic in origin is known as Nolen-Schiffer(NS) anomaly [1]. In the traditional nuclear theories origin of NS anomaly has been attributed to different aspects of theoretical charge symmetry breaking NN interaction [7] or ΛN interaction [8]. With the realization that strong nucleon–nucleon interaction has its ultimate origin in quark and gluon interaction provided by quantum chromodynamics, an alternative approach has emerged to understand the NS anomaly through incorporating explicit quark degrees of freedom in the nuclear wave functions. First few studies in this direction are of Greban and Thomas [9], Köch and Miller [10], and Wang *et al.* [11]. In [9] and [10] contribution of six quark cluster formation of the overlapping nucleons on the binding energy difference of mirror nuclei has been estimated

in the hybrid quark model [12]. In [11] calculations have been made in the framework of resonating group method. In the above studies it has been observed that quark effects make the neutron rich nucleus more bound compared to its proton rich partner and reduce the NS anomaly for lighter nuclei. Nag and Sural [13] have extended such studies to s-shell mirror hypernuclei pair ${}^4_\Lambda$He$-{}^4_\Lambda$H. Experimental binding energy of Λ-particle is more in ${}^4_\Lambda$He (proton rich partner) than in ${}^4_\Lambda$H (neutron rich partner) by nearly 360 keV. According to the calculations made in [13] quark cluster formation effects are of right sign and can account for nearly 15-20% of the NS anomaly. In the present work we have studied the effect of six quark cluster formation on the binding energy difference of lightest p-shell mirror hypernuclei pair ${}^6_\Lambda$Li$-{}^6_\Lambda$He. Our work is closely related to that of [9] and [13]. Our aim is to estimate the quark effect contribution to the binding energy difference of mirror pair ${}^6_\Lambda$Li$-{}^6_\Lambda$He and to check whether it is in the right direction to account for the observed NS anomaly. Experimental data [14–16] show that the Λ-binding energy in ${}^6_\Lambda$Li is more than in ${}^6_\Lambda$He by nearly 250 keV. Coulomb contribution, when added, will increase this discrepancy.

Our calculations are based on the hybrid quark model employed in the earlier studies. According to this model two nucleons maintain their identity as long as the distance between them is greater than a certain cutoff radius r_0. For distances smaller than r_0 the two baryons overlap and form a six quark bag. The quark contribution to the binding energy difference depends on (a) the probability of the formation of six quark bags and (b) the energy difference between the six quark bags of overlapping nucleons (or hyperon and nucleon) than that of isolated nucleons (or hyperon and nucleon). In both ${}^6_\Lambda$He and ${}^6_\Lambda$Li there is a valence nucleon outside a closed core of nucleons and Λ-particle. We have calculated the six quark bag probability of the valence particle with the core nucleon and the corresponding contribution to the binding enegy difference of the pair ${}^6_\Lambda$He and ${}^6_\Lambda$Li. We have also estimated the six quark cluster formation of the valence nucleon with the hyperon and its effect on the binding energy difference. ΛN bag formation effects are known to make significant contribution to the Λ-nonmesonic decay rates in the finite nuclei [17]. This effect was not included in the work of Nag and Sural [13]. In Section 2 we give the formalism. Calculations are discussed in Section 3 and conclusion in Section 4.

2. Formalism

Lambda particle binding energies in the ground states of ${}^6_\Lambda$Li and ${}^6_\Lambda$He hypernuclei with respect to the free particle system ${}^4_2$He$+\Lambda+n$ can be defined as (${}^5_3$Li and ${}^5_2$He are resonant states)

$$M({}^6_\Lambda\mathrm{Li}) = M({}^4_2\mathrm{He}) + m_\Lambda + m_p - \mathrm{B.E.}({}^6_\Lambda\mathrm{Li})\,, \qquad (1)$$

$$M({}^{6}_{\Lambda}\mathrm{He}) = M({}^{4}_{2}\mathrm{He}) + m_{\Lambda} + m_n - \mathrm{B.E.}({}^{6}_{\Lambda}\mathrm{He})\,. \tag{2}$$

If we neglect core contribution which is irrelevant for the binding energy difference and consider only the effect of six quark cluster formation of the valence nucleon with the core nucleon and the hyperon, an additional contribution to the binding energies of the hypernuclei results as shown below. If the probabilites that the valence nucleon forms a six quark bag with any of the core nucleons and hyperon are P^{6q}_{NN} and $P^{6q}_{\Lambda N}$ respectively and P_0 is the probability that it does not overlap with any of the nucleons or hyperon, then

$$P_0 + P^{6q}_{NN} + P^{6q}_{\Lambda N} \cong 1\,. \tag{3}$$

(The probability of formation of nine or higher quark bags is neglected). We can rewrite the mass of core plus one additonal nucleon as

$$\begin{aligned} M'({}^{6}_{\Lambda}\mathrm{Li}) &= M({}^{4}_{2}\mathrm{He}) + P_0 m_p + \frac{1}{2}P^{6q}_{NN}(m_{pp} - m_p) \\ &\quad + \frac{1}{2}P^{6q}_{NN}(m_{pn} - m_n) + m_{\Lambda} + P^{6q}_{\Lambda N}(m_{p\Lambda} - m_{\Lambda}) \\ &\quad - [(\mathrm{B.E.}({}^{6}_{\Lambda}\mathrm{Li}) + \Delta V_c({}^{6}_{\Lambda}\mathrm{Li})]\,, \end{aligned} \tag{4}$$

where m_{pp} , m_{pn} and $m_{p\Lambda}$ represent the masses of six quark bags formed of two protons, a proton and a neutron, and a proton and a hyperon. $\Delta V_c({}^{6}_{\Lambda}\mathrm{Li})$ measures how much coulomb repulsion is lost by cuttting the two proton integral off at distances $r < r_0$. Due to this the binding energy in ${}^{6}_{\Lambda}\mathrm{Li}$ increases by $\Delta V_c({}^{6}_{\Lambda}\mathrm{Li})$. We may rewrite (4) as

$$M'({}^{6}_{\Lambda}\mathrm{Li}) = M({}^{4}_{2}\mathrm{He}) + m_{\Lambda} + m_p - [\mathrm{B.E.}({}^{6}_{\Lambda}\mathrm{Li}) + \delta B({}^{6}_{\Lambda}\mathrm{Li})]\,, \tag{5}$$

where

$$\begin{aligned} \delta B({}^{6}_{\Lambda}\mathrm{Li}) &= P^{6q}_{NN} m_p + P^{6q}_{\Lambda N} m_p - \frac{1}{2}P^{6q}_{NN}(m_{pp} - m_p) \\ &\quad - \frac{1}{2}P^{6q}_{NN}(m_{pn} - m_n) - P^{6q}_{\Lambda N}(m_{p\Lambda} - m_{\Lambda}) + \Delta V_c({}^{6}_{\Lambda}\mathrm{Li})\,. \end{aligned} \tag{6}$$

Comparision of equation (5) with equation (1) shows that the six quark cluster formation of the valence nucleon with the core nucleons and the hyperon increases the binding energy of Λ-particle in ${}^{6}_{\Lambda}Li$ by $\delta B({}^{6}_{\Lambda}\mathrm{Li})$. Similarly,for ${}^{6}_{\Lambda}\mathrm{He}$ the additional contribution to the binding energy is

$$\begin{aligned} \delta B({}^{6}_{\Lambda}\mathrm{He}) &= P^{6q}_{NN} m_n + P^{6q}_{\Lambda N} m_n - \frac{1}{2}P^{6q}_{NN}(m_{np} - m_p) \\ &\quad - \frac{1}{2}P^{6q}_{NN}(m_{nn} - m_n) - P^{6q}_{\Lambda N}(m_{n\Lambda} - m_{\Lambda})\,, \end{aligned} \tag{7}$$

where m_{nn} , m_{np} and $m_{n\Lambda}$ are the masses of six quark bags of two neutrons, a neutron and a proton, and a neutron and a hyperon respectively. Concentrating on the effect of six quarks cluster formation only on the binding energy difference (denoted by ΔB_{6q}), we get

$$\begin{aligned}(\Delta B)_{6q} &= \delta B({}^6_\Lambda\mathrm{He}) - \delta B({}^6_\Lambda\mathrm{Li}) \\ &= \frac{1}{2}P^{6q}_{NN}(2m_n - 2m_p) - \frac{1}{2}P^{6q}_{NN}(m_{nn} - m_{pp}) \\ &\quad + P^{6q}_{\Lambda N}(m_{p\Lambda} - m_{n\Lambda}) + P^{6q}_{\Lambda N}(m_n - m_p) - \Delta V_c({}^6_\Lambda\mathrm{Li})\,. \qquad (8)\end{aligned}$$

In the spirit of independent particle model the ground state of the hypernucleus with $A+1$ nucleons can be written as

$$\begin{aligned}\Psi^0(1,2,\ldots A+1,\Lambda) &= \Phi_0^\Lambda \Psi_0^N \\ &= \Phi_{\alpha_0}(\Lambda)\mathcal{A}\left\{\prod_{i=1}^{A+1}\Phi_{\alpha_i}(i)\right\}, \qquad (9)\end{aligned}$$

where $\Phi_{\alpha_i}(i)$ are normalized single particle states with quantum numbers α_i,Φ_{α_0} is the hyperon state and $\mathcal{A}$ is the antisymmetrization operator.We may define a wave function

$$\Psi^v_N(1,2,\ldots A+1,\Lambda) = \Phi_{\alpha_0}(\Lambda)\mathcal{A}\left\{\prod_{\alpha_i<\alpha_v}[1-\theta(r_0 - r_{\alpha_i\alpha_v})]\prod_{i=1}^{A+1}\Phi_{\alpha_i}(i)\right\}, \qquad (10)$$

where

$$\begin{aligned}\theta(r_0 - r_{\alpha_i\alpha_v}) &= 0 \text{ for } r_{\alpha_i\alpha_v} > r_0 \\ &= 1 \text{ otherwise}.\end{aligned}$$

Ψ^v_N is written to ascertain that the valence particle in quantum state α_v does not form a six quark bag with any of the nucleons. Thus

$$P^{6q}_{NN} = \langle\Psi^0|\Psi^0\rangle - \langle\Psi^v_N|\Psi^v_N\rangle \qquad (11)$$

is the probability of the valence particle being part of one or more six quark bags with the core nucleons and gives us the amount of six quark admixture in the nuclear wave function.The overlap probability P^{6q}_{NN} can be expressed as a sum of single particle term [9] to lowest order in correlation function $\theta(r_0 - r_{ij})$ as

$$P^{6q}_{NN} = \sum_{\alpha_m=\alpha_1}^{\alpha_A} P_{\alpha_m}(r_0)\,, \qquad (12)$$

where

$$P_{\alpha_m}(r_0) = \langle \Phi_{\alpha_v}(1)\Phi_{\alpha_m}(2)|\theta(r_0 - r_{12})|\Phi_{\alpha_v}(1)\Phi_{\alpha_m}(2) - \Phi_{\alpha_m}(1)\Phi_{\alpha_v}(2)\rangle . \tag{13}$$

α_v and α_m define the quantum states of the valence and the core nucleons respectively. Equation (12) in turn can be written as

$$\begin{aligned} P^{6q}_{NN}(r_0) &= \sum_{\alpha_m=\alpha_1}^{\alpha_A} P_{\alpha_m}(r_0) \\ &= \sum_{n_i l_i j_i \tau_{z_i}} (2j_i + 1) P_{n_i l_i j_i \tau_{z_i}}(r_0) . \end{aligned} \tag{14}$$

$P_{n_i l_i j_i \tau_{z_i}}(r_0)$ can be interpreted as the probability for the valence particle to be within a distance r_0 of a specified core particle with quantum numbers $n_i l_i j_i \tau_{z_i}$ and is

$$\begin{aligned} P_{n_i l_i j_i \tau_{z_i}}(r_0) &= \frac{1}{(2j_v+1)(2j_i+1)} \\ &\times \sum_{m_v m_i} \langle \Phi_{\alpha_v}(1)\Phi_{\alpha_i}(2)|\theta(r_0 - r_{12}|\Phi_{\alpha_v}(1)\Phi_{\alpha_i}(2) - \Phi_{\alpha_i}(1)\Phi_{\alpha_v}(2)\rangle , \end{aligned} \tag{15}$$

where the sum is over the magnetic substates. If we assume that the difference between neutron and proton orbits can be ignored, the isospin index can be suppressed.Thus from equations(14) and (15) $P^{6q}_{NN}(r_0)$ can be expressed as a combination of a direct term $P^d_{n_i l_i j_i}(r_0)$ and an exchange term $P^e_{n_i l_i j_i}(r_0)$ as

$$P^{6q}_{NN}(r_0) = \frac{1}{(2j_v+1)} \sum_{n_i l_i j_i} \left[2P^d_{n_i l_i j_i}(r_0) - P^e_{n_i l_i j_i}(r_0) \right] , \tag{16}$$

where

$$P^d_{n_i l_i j_i}(r_0) = \sum_{m_v m_i} \langle \Phi_{\alpha_v}(1)\Phi_{\alpha_i}(2)|\theta(r_0 - r_{12})|\Phi_{\alpha_v}(1)\Phi_{\alpha_i}(2)\rangle \tag{17}$$

and

$$P^e_{n_i l_i j_i}(r_0) = \sum_{m_v m_i} \langle \Phi_{\alpha_v}(1)\Phi_{\alpha_i}(2)|\theta(r_0 - r_{12})|\Phi_{\alpha_i}(1)\Phi_{\alpha_v}(2)\rangle . \tag{18}$$

In equation (16), factor 2 in the $P^d_{n_il_ij_i}(r_0)$ stems from identical proton and neutron direct contribution. Similarly

$$P^{6q}_{\Lambda N}(r_0) = P'_{n_0l_0j_0}(r_0)$$

with

$$\begin{aligned} P'_{n_0l_0j_0}(r_0) &= \frac{1}{(2j_v+1)(2j_0+1)} \\ &\times \sum_{m_0m_v} \langle \Phi_{\alpha_0}(1)\Phi_{\alpha_v}(2)|\theta(r_0 - r_{12}|\Phi_{\alpha_0}(1)\Phi_{\alpha_v}(2)\rangle . \quad (19) \end{aligned}$$

There is no exchange term for ΛN overlapping. Φ_{α_0} is the single particle orbital for hyperon. Our calculations for $P^{6q}_{NN}(r_0)$ and $P^{6q}_{\Lambda N}(r_0)$ are based on equations (14) to (19).

3. Calculations

To calculate the six quark probability $P^{6q}_{NN}(r_0)$ we have used the harmonic oscillator wave function with a uniform oscillator constant $\mathcal{V} = 0.41\mathrm{fm}^{-2}$ for 1s and 1p nucleons. This value of $\mathcal{V}$ has been fixed in the study of 1s shell hypernuclei by Gal *et al.* [18] and gives a good fit to the experimental value of charge radii measured from charge scattering experiment [19]. Using standard techniques of angular momentum algebra matrix elements in equation (15) are transformed to relative and centre-of-mass basis, the transformation coefficients being Moshinsky brackets [20,21]. The final expression for the direct term in equation (16) simplifies to

$$\begin{aligned} P^d_{n_il_ij_i}(r_0) &= \sum_{\substack{\lambda S n l N L \\ J M}} A^2 \begin{bmatrix} l_i & 1/2 & j_i \\ l_v & 1/2 & j_v \\ \lambda & S & J \end{bmatrix} \\ &\times \langle nlNL, \lambda | n_il_in_vl_v; \lambda\rangle^2_{MB} \langle nl||\theta(r_0 - r)||nl\rangle , \quad (20) \end{aligned}$$

where

$$\langle nl||\theta(r_0 - r)||nl\rangle = \int_0^{r_0} R^2_{nl}(r)dr ,$$

$R_{nl}(r)$ are normalized radial functions. The quantum numbers $nlSJ$ and NL refer to the relative and center-of-mass (CM) state of the overlapping pair respectively. The angular momentum J is the result of coupling l and S and λ is the result of coupling l and L. $\langle nlNL, \lambda | n_il_in_vl_v; \lambda\rangle_{MB}$ are the

Moshinsky brackets. The expression for the exchange term is similar to (20) with an additional phase factor of $(-1)^{\lambda+S+l_i+l_v+1}$. In the calculation of $P_{\Lambda N}^{6q}(r_0)$, to facilitate Moshinsky transformation to relative basis in the matrix elements in equation (19), we have chosen $\mathcal{V}_\Lambda = (m_\Lambda/m_N)\mathcal{V}_N$. With $\mathcal{V}_N = 0.41\text{fm}^{-2}$, $\mathcal{V}_\Lambda$ is fixed at 0.49fm^{-2}. This prescription has been used earlier by others in the study of hypernuclei [22, 23]. With this ansatz matrix elements in equation (19) can be expanded in relative and center-of-mass basis, the expansion coefficients being Smirnov coefficients. Thus equation (19) can also be simplified to an expression similar to (20) with Moshinsky bracket replaced by Smirnov bracket $\langle nlNL,\lambda|n_0l_0n_vl_v,\lambda\rangle_{SM}$ [24]. The Coulomb energy difference of ${}^6_\Lambda$Li and ${}^6_\Lambda$He is

$$V_c = \frac{(2j_i+1)}{(2j_v+1)} \sum_{\substack{\lambda SnlNL \\ JM}} A^2 \begin{bmatrix} l_i & 1/2 & j_i \\ l_v & 1/2 & j_v \\ \lambda & S & J \end{bmatrix} \times \langle nlNL;\lambda|n_il_in_vl_v;\lambda\rangle_{MB}^2 \langle nl||v_c||nl\rangle\,, \tag{21}$$

where $v_c = e^2(1+2\tau_{z_i})/4r$ is the Couloumb interaction between the valence proton and the core protons. The loss in Coulomb energy ΔV_c when the two protons overlap for $r < r_0$ can be obtained from an expression similar to (20) with the radial matrix element equal to $\langle nl||\theta(r_0-r)v_c||nl\rangle$. The difference of six quark clusters for two neutrons m_{nn} and two protons m_{pp} has been estimated by Köch and Miller [10] both in the nonrelativistic quark model (NRQM) and in MIT bag model. In the case of NRQM, these authors have used three different sets for oscillator length parameter and strong interaction parameter α_s , and have obtained slightly different values for $m_{nn} - m_{pp}$. The mass difference of six quark cluster of neutron hyperon $m_{n\Lambda}$ and proton hyperon $m_{p\Lambda}$ has been calculated by us in the following manner. The mass of $3n$ quark bag can be expressed as [9]

$$E = 1.44 \sum_{i<j}^{3n} \frac{Q_iQ_j}{R_{3n}} + 0.42 \sum_{i=1}^{3n} \frac{C_i}{R_{3n}} \tag{22}$$

if the terms which do not contribute to the mass difference are omitted and $m_i = C_i/R$. Q_i and m_i are the charge and the mass of the i^{th} quark respectively and R_{3n} is the radius of $3n$ quark bag. With $C_d - C_u = 4$ MeV and $R_6 = 1.2$fm equation (22) leads to a difference of $m_{p\Lambda} - m_{n\Lambda} = 0.599$ MeV. The six quark probabilities $P_{NN}^{6q}(r_0)$ and $P_{\Lambda N}^{6q}(r_0)$ depend on the cutoff radius r_0. If nucleons have radii of about 1 fm, r_0 is expected to be of the same order. If r_0 is much smaller than 1 fm, P^{6q} becomes very small. If $r_0 >$ 1fm the conventional meson exchange picture of nuclear forces is

TABLE I

Probabilities $P^{6q}_{NN}(r_0)$ for the valence nucleon to form a six quark bag with the core nucleons as a function of cut off radius r_0.

r_0(fm)	Direct term	Exchange term	$P^{6q}_{NN}(r_0)$
0.85	0.0316	0.0135	0.0124
0.89	0.0418	0.0179	0.0164
0.93	0.0546	0.0234	0.0215
0.95	0.0549	0.0235	0.0216
1.0	0.0706	0.0303	0.0278
1.1	0.1382	0.0592	0.0543

difficult to understand. In most of the earlier studies [10, 13, 17] a value of $r_0 = 1$fm has been preferred. In Table I and Table II we present the results for $P^{6q}_{NN}(r_0)$ and $P^{6q}_{\Lambda N}(r_0)$ for r_0 ranging between 0.85 fm to 1.1 fm. It is worth noting that the Pauli exchange term in $P^{6q}_{NN}(r_0)$ is about 40% of the direct term and leads to a sizable reduction in the six quark probability. Using the value calculated by us of $P^{6q}_{NN}(r_0)$ and $P^{6q}_{\Lambda N}(r_0)$, $m_{n\Lambda} - m_{p\Lambda}$, ΔV_c and NRQM and MIT valuse of $m_{nn} - m_{pp}$ we have estimated the six

TABLE II

Probabilities $P^{6q}_{\Lambda N}(r_0)$ for the valence nucleon to form a six quark bag with the hyperon as a function of r_0.

r_0(fm)	$P^{6q}_{\Lambda N}(r_0)$
0.85	0.0035
0.89	0.0047
0.93	0.0061
0.95	0.0062
1.0	0.0080
1.1	0.0159

quark cluster contribution to the binding energy difference between ${}^6_\Lambda$Li and ${}^6_\Lambda$He for different cases. In Table III we present the result for $r_0 = 1.0$fm for various terms. We can compare our value of $P^{6q}_{NN}(r_0) = 0.028$ with the value of $P^{6q}_{NN}(r_0) = 0.0342$ obtained by Nag and Sural for $A = 4$ hypernuclei for same r_0. As expected our value is slightly smaller because the valence nucleon in $A = 6$ hypernuclei is in the relative p-state and is expected to have smaller overlap with core nucleons compared to the overlap between s-state

TABLE III

Six quark cluster effect on the Λ-binding energy difference between mirrer hypernuclei ${}^{6}_{\Lambda}$He and ${}^{6}_{\Lambda}$Li for the cutoff radius r_0 = 1fm. The values of ΔB_{6q} obtained by us are shown in the last column of the Table. Values of other quantities mentioned in the text are shown in other columns.

Model	$m_{nn}-m_{pp}$ (MeV)	$P^{6q}_{NN}(r_0)$	$P^{6q}_{\Lambda N}(r_0)$	ΔV_c (MeV)	$m_{p\Lambda}-m_{n\Lambda}$ (MeV)	$(\Delta B)_{6q}$ (keV)
NRQM 1	-1.86					42
NRQM 2	-1.45	0.028	0.008	0.035	0.599	36
NRQM 3	-0.58					24
MIT Bag	0.432					10

nucleons as in $A = 4$ hypernuclei. The largest contribution to the binding energy difference is obtained as 42 keV for NRQM Model 1 out of which 15 keV is obtained from the hyperon nucleon cluster formation probability.

Charge symmetry of the ΛN interaction leads to the expectation that Λ-hyperon should have equal binding energy in mirror pair ${}^{6}_{\Lambda}$He $-{}^{6}_{\Lambda}$ Li. The experimental Λ-binding energy in ${}^{6}_{\Lambda}$He and ${}^{6}_{\Lambda}$Li are 4.25 MeV [14, 15], and 4.50 MeV [15, 16], respectively showing a difference of 250 keV. The experimental value of Coulomb displacement energy for the mirror pair ${}^{6}_{\Lambda}$He $-{}^{6}_{\Lambda}$ Li is not available. If we add our calculated value of 468 keV for r_0 = 1fm the discrepancy increases to a substantial value of 718 keV indicating a large violation of charge symmetry breaking effect. The results of our calculations show that six quark cluster formation effect increases the binding of Λ-hyperon in ${}^{6}_{\Lambda}$He than in ${}^{6}_{\Lambda}$Li. This is in accordance with the observation made in [9] and [10] that quark effects increase the binding of a neutron rich nucleus in comparison to that of its proton rich partner because the colour magnetic hyperfine interaction between quarks make $m_{nn} - m_{pp}$ less than the corresponding term $2m_n - 2m_p$ for free nucleons. This however does not give the right sign to the NS anomaly for ${}^{6}_{\Lambda}$Li$-{}^{6}_{\Lambda}$He pair. It is worth mentioning that the right sign obtained in [13] for ${}^{4}_{\Lambda}$He$-{}^{4}_{\Lambda}$H is probably due to the particular values of P_{6q} and $\tilde{P}_{6q}$ (defined in [13]) used in the calculation.Only P_{6q} is related to experimental data but $\tilde{P}_{6q}$ is strongly dependent on the wave functions and the other parameters used in the calculation.

4. Conclusion

We have made a simple calculation of the six quark effect contribution to the binding energy difference of the mirror pair hypernuclei ${}^{6}_{\Lambda}$He $-{}^{6}_{\Lambda}$ Li. The calculated values are small (42 keV to 24 keV for NRQM and 10 keV for bag model) and depend very much on the model used. The overlap probability of

the valence nucleon with the hyperon also make a small contribution to the binding energy difference and should be included in any reliable estimate. However, quark effects make Λ-hyperon more bound in ${}^{6}_{\Lambda}$He than in ${}^{6}_{\Lambda}$Li and do not give correct sign to the NS anomaly. Unfortunatly the nuclear wave functions needed as input to determine the matrix element in the calculation of P^{6q} are not directly related to experiments as in the case for $A = 3$ nuclei. We probably need a detailed description of the conventional wave function and a better study of various other contributions to the binding energy difference to understand the large charge symmetry violation in the experimental results.

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The Schmidt Diagrams and Quark Degree of Freedom

By

Madhulika Mehrotra And Indira Mehrotra

Department Of Physics, University Of Allahabad,
Allahabad-211002,India

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The magnetic moments of nuclei with one nucleon outside a closed core are given by Schmidt line in the shell model framework,but the experimental values differ from the single particle values. To understand the deviation of the magnetic moments of nuclei from the Schmidt values is one of the fundamental problems of nuclear physics. Several authors have tried to explain this deviation by including various corrections like configuration mixing effect [1-3], effective moment operator [4,5], exchange current [6.7] etc. With the realisation that quark effects in nuclei play an important role in understanding

several nuclear phenomenon Karl et al [8] and independently Radhey Shyam et al [9] have assigned quark degrees of freedom to the nucleons to explain the observed magnetic moments of A=3 nuclei and of deuteron respectively.

In the present work we have estimated the quark contribution to the magnetic moments of mirror nuclei with A = 13,17,29,33,41. Our calculations are based on a hybrid quark model according to which two overlapping nucleons form a six quark bag if their relative separation distance is less than a certain critical radius $r_0 = 1$ fm. In the bag model the magnetic moment of a quark is proportional to the radius of the bag. When the number of quarks in a bag is increased from 3 to 6 its radius is increased by a factor of about 4/3. Thus in the quark picture the magnetic moment of a nucleon inside a six quark bag is larger by about a factor 4/3. In the hybrid quark model framework the magnetic moments of mirror nuclei can be expressed as, for neutron

$$\mu_{6q} = \pm\frac{2J}{(2l+1)}\Big[(1-P^{6q}_{NN})\mu_n + P^{6q}_{NN}\mu_n'\Big] \Bigg|\begin{matrix} J=l+\frac{1}{2} \\ J=l-\frac{1}{2}\end{matrix}$$

for proton

$$\mu_{6q} = J\Big[1\pm\Big(\frac{(-1)}{(2l+1)}\Big)\Big] \pm \frac{2J}{(2l+1)}\Big[(1-P^{6q}_{NN})\mu_p + P^{6q}_{NN}\mu_p'\Big] \Bigg|\begin{matrix} J=l+\frac{1}{2} \\ J=l-\frac{1}{2}\end{matrix}$$

P^{6q}_{NN} is the probability when the valence nucleon forms a six quark bag with any of the core nucleons. $\mu_n'(\mu_p')$ are the magnetic moments of neutron(proton) inside a six quark bag. The results of our calculation are shown in Table 1. P^{6q}_{NN} valuse are taken from the work of Koch and Miller [10]. We find that quark bag formation effect contributes significantly and tend to increase the magnetic moment of nuclei from their single particle values. In some of the

cases our estimated values are closer to the experimental values. In last column of Table 1 we have given the corrections to the single particle values due to various other effects as estimated by different authors. The results of Table 1 suggests that quark effect corrections be considered alongside other corrections in accounting for deviations from Schmidt values.